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Pareto-improving Consumption Tax When the Return from Capital is idyosyncratic and (Optimal or non-Optimal) Capital Income Tax is available

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Pareto-improving Consumption Tax When the Return from Capital is idyosyncratic and (Optimal or non-Optimal)Capital Income Tax is available

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Abstract

In the standard multi-period model, the consumption tax and the wage tax are equivalent. When a capital market is incomplete, such that the rate of return from capital is idiosyncratic, the consumption tax, in contrast to the wage tax, can play a role in risk-sharing. However, risk-sharing may disappear if the government applies a linear or non-linear capital income tax, because the source of risk is the return from capital. The present study shows that a consumption tax, when instituted in the presence of a wage tax, increases welfare when the capital market is incomplete, even if the government applies a non-linear capital income tax for risk-sharing and subsidizes investments.

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1 Introduction

Revenue derived from the consumption tax is substantial, and in some cases it is nearly the same as revenue derived from direct income taxes in many developed countries. Accordingly, it would be of interest to determine the optimal ratio of consumption tax to income tax, from the perspective of economic efficiency. In a simple standard multi-period model, the consumption tax and the wage tax induce exactly the same consumption and labor-supply behavior and generate the same discounted value of tax revenue, as long as the wedge caused by the two taxes is the same. Although the two taxes induce different personal-savings behaviors, because of their different effects on the timing of taxation, the consumption tax and the wage tax induce the same resource allocation if government savings are taken into consideration. This means that the consumption tax becomes redundant in the presence of the wage tax. Thus, a simple multi-period model cannot explain the optimal ratio of consumption and wage taxes.[1]

The optimal ratio between these two forms of tax can be evaluated with respect to the liquidity constraints of the economy. Judd and Hubbard (1987) analyzed the impact of fiscal policy under liquidity constraints. When consumers face liquidity constraints due to the imperfection of financial markets, they prefer to pay to tax in later periods than earlier periods. This implies that the consumption tax is more efficient than the wage tax since the tax liability of the wage tax occurs only when consumers are working while the tax liability of the consumption tax is spread

[1]This conclusion does not change even in the presence of a non-linear income tax system. In the famous papers of Atkinson and Stiglitz (1976, 1980), it was shown that the introduction of price distortions between consumer goods is optimal if the preference between consumption goods and leisure is not weakly separable in the presence of a non-linear income tax. However, consumption tax does not distort the relative price of consumption over multiple periods. Thus, although the Atkinson and Stiglitz theorem is appropriate to explain the use of capital income tax, it does not address the optimal combination of consumption and wage taxes.

over the consumer's life-time.

Tax-evasion behavior also offers insight into the optimal tax ratio. Boadway, Marchand and Pestieau (1994) showed that, in the presence of tax-evasion behavior affecting labor-derived income tax, an optimal combination of consumption tax and labor income tax is important.

The present study demonstrates that when a capital market is incomplete, such that the rate of return from investment is idiosyncratic among individuals and that there is no insurance to insure stochastic returns, the introduction of a consumption tax, even in the presence of a wage tax, can increase efficiency although the government may also be using a capital income tax to absorb idiosyncratic risks.

The importance of an incomplete capital market can be seen in the data in several ways. First, the share of non-financial asset, whose stochastic rate of return are not likely to be insured, is not low. For example, Bucks, Kennickell and Moore (2006) reported that, according to the US Survey of Consumer Finance 2004, only 35.7 percent of all assets are held in the form of publicly traded financial assets while the rest are held as non-financial assets, such as self-owned businesses, self-occupied homes, and real estate. Wolf (2002) also reported that for top 20 percent of households in the Survey of Consumer Finances 1998, the share of residential homes, unincorporated businesses and real estate in the total assets is 52 percent. It should be noted that the share of publicly traded financial assets becomes much lower if human capital, which is not included in the Survey of Consumer Finances, is taken to be a component of consumer assets. Generally, there is no insurance that insures the capital gains/losses of the value of residential home, real estate, unincorporated business and human capital due to moral hazard and adverse selection.

This implies that consumers buying those assets can be facing substantial risks, presumably with the trade off of higher expected return.

Second, the variation of the rate of return of those assets is not trivial. Figure 1 and 3 show the distribution of the rate of return from self-owned business and primary residential home, which is calculated from the Survey of Consumer Finances 1983-1989. As two graphs of the distribution of the rate of return show, the rate of return varies substantially across individuals. But one might argue that the substantial variation of the rate of return from those investments is due to the heterogeneity of risk preference across individuals rather than idiosyncratic shocks. To control preference heterogeneity of consumers, I regressed the rate of return on demographic factors such as age and education and plotted the residuals in Figure 2 and 4. Those two graphs show that, even after controlling the demographic factors that account for the heterogeneity of preference, the variation of the rate of return is substantial.

Although it is still possible to argue that the distribution of the residual rate of return still reflects heterogeneity of unobserved risk preference, which cannot be controlled by the demographic factors, general anecdotal evidences also indicate that substantial part of the variation of the rate of return comes from real idiosyncratic shocks and those shocks are not insured.

The ability of a consumption tax to increase efficiency when the insurance market for investors is incomplete can be understood by examining the basic economic logic behind the equivalence of the consumption tax and the wage tax. The key observation in this respect is that, in a simple multi-period model, the budget constraint of each period can be merged into an intertemporal budget constraint by eliminating saving variables. As a result, individual decision-making can be defined as maximizing life-time utility, subject to an intertemporal budget constraint. In this case, the intertemporal budget constraint together with the consumption tax can be re-written as another budget constraint that also includes the wage tax, by dividing the intertemporal budget constraint by one plus the consumption tax rate. This implies that, as long as the tax wedge of these two taxes is the same, the consumption tax and the wage tax induce the same allocation.

This logic does not change even when there is uncertainty, as long as the market can issue state-contingent securities. For example, consider a two-period model in which there is uncertainty in the second period. When the market is able to issue state-contingent security, the budget constraints of the two periods and of all states can still be merged into a single budget constraint. Thus, the above logic can still be applied.

However, when the market is incomplete, budget constraints cannot be merged. In that case, individual decision-making cannot be characterized as maximizing life-time utility subject to a single budget constraint. Thus, the logic supporting the equivalence of the two taxes cannot be applied to a model that includes incomplete markets.

In the real world, there are many uncertain events for which state-contingent security is not available and the consumption tax could play an important role in risk-sharing. One example is investment in local real estate, local business, human capital or owner-occupied housing. Generally, the rate of return from those investments is different among individuals ex-post, but there is no insurance market that insures the stochastic return from those investments.

The basic idea of this paper is that the consumption tax may be able to help the government share those risks that cannot be shared by the market. To illustrate, consider the case in which the return from investment is stochastic due to the incompleteness of the market. Assume that there are two periods and that an individual works in the first period and she/he consumes in both the first and the second periods. Also, assume that there is an investment opportunity for each individual and the return from this investment opportunity is stochastic. The second assumption implies that consumption in the second period is also stochastic. When the government depends entirely on the income tax from labor, government revenue comes only from the first period and, as result, the government revenue from an individual is non-stochastic. However, when the government relies entirely on the consumption tax, the government revenue from one individual becomes stochastic. This implies that the government shares in some of the risk of the economy. In that case, the consumption tax acts as a form of social insurance and it increases economic efficiency unless the stochastic rate of returns that individuals invest are perfectly correlated.

Two questions arise from the above argument, however. The first is whether the private insurance market is able to insure individual investment risks and whether social insurance through consumption tax is redundant. The second is whether social insurance through a consumption tax is redundant when the government can instead use a (non-linear) capital income tax, because the source of risk is the return from capital and the non-linear capital income tax can absorb such risks.

To answer the first question, it is assumed that the capital market for investors is incomplete in the sense that there is no insurance to insure the stochastic rate of return from investment. There are two justifications for this assumption. First, empirically, the rate of return from an investment is not the same among individuals. If all risks are insured, the rate of return from an investment should be the same for all individuals. Second, to insure this type of investment risk would involve both a moral hazard problem and an adverse selection problem. For example, if there is no pressure to find a profitable investment opportunity because the insurance market insures investment risks, people will not work hard to invest profitably. As for the second question, it is addressed here in detail and is the focus of this paper.

The idea that taxation acts as a form of insurance is not new in the literature. Many authors have pointed out this effect. For example, Eaton and Rosen (1980) showed that the wage tax acts as insurance when the return from human capital is uncertain. Kopczuk (2006) demonstrated that inheritance taxation provides social security for the rich. Krueger (2003) calculated the optimal tax progression, focusing on the social-insurance role of taxation. Diamond (2003) analyzed various social insurance and tax policies with respect to life-cycle when the market is incomplete. The contribution of the present paper is the demonstration that the consumption tax increases efficiency even if the government has instituted a non-linear capital income tax to absorb risks arising from the idiosyncratic rate of return from capital.

The organization of this paper is as follows. In section 2.1, it is shown that an introduction of the consumption tax in the presence of the wage tax is welfare-improving. In section 2.2,this conclusion is shown to hold even if the government uses a linear capital income tax. In section 2.3, the result is demonstrated to be robust even if it is assumed that the government has instituted a non-linear capital income tax. In section 3, the conclusions are presented and issues for future research are discussed.

2 Analysis

2.1 A simple illustration

Before offering a detailed model, this subsection demonstrates the effect on welfare of switching from the wage tax to the consumption tax when a capital market is incomplete. Consider an economy in which there is a continuum of agents whose preferences are identical and who live for two periods. Those agents work only in the first period and they save for consumption in the second period. Each individual allocates his or her savings by investing in one safe asset and one risky asset. In this economy, technology is linear and the rate of return from the safe asset is r and the stochastic rate of return from the risky asset is $\widetilde{r}$. In this paper, $\widetilde{}$ is used to denote that the variable with $\widetilde{}$ is a random variable. It is assumed that $\widetilde{r} = \varepsilon \times (e+1)$, where ε is a random variable and $e \in [0, 1]$ is the individual's effort level in searching for profitable investment opportunities. It is also assumed that the random variable ε is independently and identically distributed (i.i.d.) across agents and that the level of individual effort to look for a profitable investment opportunity is not observable by the public. This lack of observability prevents private financial institutions from sharing in investment risks. In addition, throughout this paper, the assumptions have been made that the expected rate of return from risky investments is always greater than the rate of return from the safe asset, as long as the agent makes a positive effort to look for a profitable investments opportunities and that the expected rate of return from those investments is equal to that of the safe asset if no such effort is made. Let $E[\widetilde{r}|e]$ be $\mu(e)$, where $E[\cdot|e]$ is the operator to calculate the mathematical expectation for a given effort level e. Then, our assumption implies that

$$\mu(e) > r \qquad \text{for any level of effort } e \in (0,1] \tag{1}$$

$$\mu(e) = r \qquad \text{for } e = 0 \tag{2}$$

Thus, the agent can always invest in the safe asset with the rate of return r without any effort.

The preference of an agent is represented by the following additive and separable utility function:

$$u(c_1) - h(l) - v(e) + \frac{1}{1+\rho} E[u(\widetilde{c}_2)] \tag{3}$$

where $u' > 0$, $u'' < 0$, $h' > 0$, $h'' > 0, v' > 0, v'' > 0$

where (l, c_1, c_2) are the labor supply, the consumption in period 1 and the consumption in period 2. It is assumed that the consumption in period 1 is a normal good. From the linear technology assumption, the rate of return from investment and the wage rate are independent of labor supply and the amount of investment.

As for government, it is assumed that the government can institute a linear consumption tax and a linear wage tax.[2] In this subsection, I assume that the government does not impose a capital income tax. In the next two subsection, I analyze two cases where the government can

[2]The assumptions that the government institutes only a linear consumption tax and only a linear wage tax are consistent with the literature on Ramsey-type optimal taxation. In this literature, it is assumed that the government cannot directly control labor supply and investments, although it is aware of consumer preferences and the technology of the economy. The assumption about the government's inability to directly control either the labor supply or consumption can be easily justified by using the framework developed by Mirrlees (1971) and by assuming a model in which there are multi-agents with unobserved abilities. However, to simplify the analysis, it is assumed that the government uses a linear consumption tax, a linear wage tax, and either a quasi-linear or a non-linear capital income tax.

use a linear capital income tax and a more flexible non-linear capital income tax, respectively.[3] Let s and I be the amount of investment for the safe asset and the risky asset, respectively. Let t_w be the rate of the wage tax, t_c the rate of the consumption tax. Then, for the given wage tax rate t_w and the consumption tax rate t_c , each agent will solve the following optimization problem:

$$\max_{s,l,e} \; u(\frac{(1-t_w)wl - s - I}{(1+t_c)}) - h(l) - v(e) + \frac{1}{1+\rho} E[u(\frac{s(1+r) + I(1+\widetilde{r})}{(1+t_c)})] \tag{4}$$

If an interior solution is assumed, then the first-order conditions of l,s, I and e are

$$\frac{(1-t_w)w}{(1+t_c)} u'(c_1) = h'(l), \tag{5}$$

$$u'(c_1) = \frac{1+r}{1+\rho} E[u'(\widetilde{c}_2)] \tag{6}$$

$$u'(c_1) = \frac{1}{1+\rho} E[u'(\widetilde{c}_2)(1+\widetilde{r})] \tag{7}$$

$$\text{and } v'(e) = \frac{1}{1+\rho} E[u'(\widetilde{c}_2) \frac{d\widetilde{r}}{de} \frac{I}{1+t_c}] \tag{8}$$

$$\text{where } c_1 = \frac{(1-t_w)wl - s - I}{(1+t_c)}) \text{ and } \widetilde{c}_2 = \frac{s(1+r) + I(1+\widetilde{r})}{(1+t_c)}$$

These first-order conditions result in the following relationship between the expected rates of return of the risky investment and the safe asset.

$$-cov(u'(\widetilde{c}_2), \widetilde{r}) = E[u'(\widetilde{c}_2)]\{E[\widetilde{r} \,|e^*] - r\} \tag{9}$$

[3]It is assumed that the consumption tax rate is constant over time. In the next two subsections, I allow a linear capital income tax and a non-linear capital income tax. With capital income taxation, the assumption of time-constant consumption tax is not restrictive since time-varying consumption tax can be decomposed into time-constant consumption tax and capital income tax.

Note that $\widetilde{r}$ and $\widetilde{c}_2$ are positively correlated. Thus, we must have $E[\widetilde{r}|e^*] - r > 0$ and this requirement is satisfied by assumption(1). In other words, as long as an agent makes a positive investment in the risky asset, the expected rate of return from that investment must be greater than the rate of return from the safe asset.

Now consider the effects of the consumption tax and the wage tax. From the above-described first-order conditions, it is clear that neither the consumption tax nor the wage tax affect intertemporal conditions. Instead, the two taxes change only the relative price of consumption and leisure. Thus, two situations can be considered: (1) the government imposes only the wage tax, and (2) the government imposes only the consumption tax. Furthermore, it is assumed that in both cases the real wage rate, $w(1-t_w)/(1+t_c)$, is the same. Let $\{\widehat{l}, \widehat{e}, \widehat{s}, \widehat{I}; \widehat{c}_1, \widehat{\widetilde{c}}_2, \}$ be the solution of the above optimization problem and resulting consumption in periods 1 and 2 with $(t_w, t_c) = (\tau, 0)$. Also let and $\{l^*, e^*, s^*, I^*; c_1^*, \widetilde{c}_2^*, \}$ be the solution and resulting consumption of periods 1 and 2 with $(t_w, t_c) = (0, \tau/(1-\tau))$. Note that in either case the real wage, $w(1-t_w)/(1+t_c)$, is equal to $w(1-\tau)$. From the above first order conditions, if $\{\widehat{l}, \widehat{e}, \widehat{s}, \widehat{I}; \widehat{c}_1, \widehat{\widetilde{c}}_2, \}$ is the solution with the taxes $(t_w, t_c) = (\tau, 0)$, then $l^* = \widehat{l}, e^* = \widehat{e}, c_1^* = \widehat{c}_1, \widetilde{c}_2^* = \widehat{\widetilde{c}}_2, s^* = \widehat{s}(1+t_c), I^* = \widehat{I}(1+t_c)$.[4]

In other words, when the government switches from the wage tax to the consumption tax, consumers' labor supply and efforts are the same as before and individuals increase their investments in safe and risky assets $1 + t_c$ times. Thus, the two cases induce exactly the same l,e,c_1 and $\widetilde{c}_2$. This implies that the expected utilities of the agents under the two tax regimes are the exactly the same as long as the real wage rate, $w(1-t_w)/(1+t_c)$, is the same. When there is

[4]Note that from the first order condition of e and $I^* = \widehat{I}(1+t_c)$, we will have $e^* = \widehat{e}$.

no uncertainty and, as a result, the $\mu(e)$ is equal to r, it is straightforward to show that the tax revenue in both cases are the same. Thus, two taxes induce the same resource allocation. This is the standard argument to support the equivalence of the consumption tax and the wage tax.

However, when there is uncertainty and the capital market is incomplete, the tax revenue of two cases are different. For $(t_w, t_c) = (\tau, 0)$, government revenue is:

$$\text{Tax revenue} = \tau w \widehat{l} \tag{10}$$

In the second case, in which $(t_w, t_c) = (0, \tau/(1-\tau))$, the present value of the tax revenue is $t_c c_1^* + \frac{1}{1+r} E[t_c \widetilde{c}_2^*]$. Note that for the second period tax revenue, the government discounts it with $1+r$ since there is no macroeconomic shock regarding both capital income and consumption in the second period. Also note that $c_1^*(1+t_c) = wl^* - s^* - I^*$ and $(1+t_c)\widetilde{c}_2^* = s^*(1+r) + I^*(1+E[\widetilde{r}|e^*])$. Therefore, the tax revenue under the consumption tax is:

$$\text{Tax revenue} = \frac{t_c}{(1+t_c)}(wl^* - s^* - I^*) + \frac{1}{(1+r)}\frac{t_c}{(1+t_c)}\{s^*(1+r) + I^*(1+E[\widetilde{r}|e^*])\} \tag{11}$$

$$= \tau w l^* + \tau I^* \{\frac{1+E[\widetilde{r}|e^*]}{1+r} - 1\} \tag{12}$$

Note that $l^* = \widehat{l}$ and $E[\widetilde{r}|e^*] > r$. Accordingly, the tax revenue generated by the consumption tax alone is more efficient than that generated by the wage tax alone. Thus, as long as individuals positively invest in risky assets, the consumption tax is more efficient. The intuition for this result is quite simple: when the government uses only the wage tax, government revenue from one individual is certain. But when the government switches to the consumption tax, the tax revenue from one individual becomes stochastic while individual consumption and the labor supply remain the same as before. This implies that, under the regime of the consumption tax, the government starts to share some of the risk that individuals initially face. Since individual risks are not perfectly correlated, risk-sharing through the consumption tax increases economic efficiency.

However, there is another issue that needs more careful discussion regarding the above argument which is as follows: Uncertainty regarding the economy comes from uncertainty in the rate of return from risky investments. Thus, one might argue that the government can institute a capital income tax to absorb the risk of the stochastic rate of return from risky investments. If the government institutes a non-linear capital income tax to absorb those risks, risk-sharing through the consumption tax may be redundant. To address this criticism, it is clear that a more formal analysis is needed. Thus, in the next section, the effects of applying a consumption tax in the presence of a wage tax and either a linear or non-linear capital income tax are examined in the context of a two-period model.

2.2 A model with a linear capital income tax

It was shown above that the consumption tax can help in risk-sharing in an economy in which there are financial risks that the market cannot insure. However, one might speculate that a consumption tax will be redundant in the presence of a capital income tax because the source of risks is the return on investment and because a capital income tax can absorb such risks. Here, it will be shown that this result does not change even if the government uses a linear tax (tax liability from random capital income is proportional to capital income plus some constant).

Let t_{ks} be the tax rate of capital income from safe assets and t_{kr} be the tax rate on capital income from risky assets. Let D be the constant that makes the tax liability on risky capital income affine, and t_I the subsidy rate on risky investments. [5] In other words, the tax liability from risky investments is $t_{kr}(\widetilde{r}I) - D$. Accordingly, the agent will maximize the following utility function:

$$\max_{\{l,s,I,e\}} u(\frac{wl(1-t_w)}{1+t_c} - \frac{s}{1+t_c} - \frac{(1-t_I)I}{1+t_c}) - h(l) - v(e) + \frac{1}{1+\rho} E[u(\frac{(1+r)s - t_{ks}rs + (1+\widetilde{r})I - t_{kr}\widetilde{r}I + D}{1+t_c})] \tag{13}$$

Now let $w(1-t_w)/(1+t_c)$ and $D/(1+t_c)$, $s/(1+t_c)$, $I/(1+t_c)$ be $\overline{w}$ and $\overline{D}$, $\overline{s}$ and $\overline{I}$, respectively. Then, the above optimization problem can therefore be rewritten as follows:

$$\begin{aligned} V(\overline{w}, \overline{D}, t_{ks}, t_{kr}, t_I; t_c) \equiv \max_{\{l,e,\overline{s},\overline{I}\}} & u(\overline{w}l - \overline{s} - (1-t_I)\overline{I}) - h(l) - v(e) \\ & + \frac{1}{1+\rho} E[u(\ \overline{s}(1 + r(1-t_{ks})) + (1+\widetilde{r}(1-t_{kr}))\overline{I} + \overline{D})] \end{aligned} \tag{14}$$

The optimization problem (14) has an important implication. (14) shows that as long as $\overline{w}$ and $\overline{D}$ do not change, a change of the consumption tax rate does not change the optimized value of (14) because t_c does not directly enter (14). On the other hand, the government can control $\overline{w}$ and $\overline{D}$ by controlling t_w and D directly. This implies that the government can have policy instruments to offset the effect of the consumption tax. Let the solution of (14) be $(l^*, e^*, \overline{s}^*, \overline{I}^*)$. Also let the resulting consumption in period 1 and period 2 be c_1^* and $\widetilde{c}_2^*$. Since the maximized value does not directly depend on t_c once $\overline{w}, \overline{D}$ are defined, we have

[5] The availability of the subsidy on risky investments does not change the result. I allow the subsidy on investment to demonstrate that the result is robust. Even if the subsidy on risky investments are not available, the result does not change.

$$\partial V(\overline{w}, \overline{D}, t_{ks}, t_{kr}, t_I; t_c)/\partial t_c = 0 \tag{15}$$

$$\partial l^*/\partial t_c = 0, \partial e^*/\partial t_c = 0, \partial \overline{s}^*/\partial t_c = 0, \partial \overline{I}^*/\partial t_c = 0 \tag{16}$$

Then, the government's problem is to choose $t_w, D, t_{ks}, t_{kr}, t_I$ and t_c in order to maximize the value function subject to the government budget constraint. Let $\overline{Q}$ be the required government revenue. The government budget constraint is

$$t_w wl + \frac{t_{ks} rs + t_{kr}\mu I - D}{1+r} + t_c c_1 + \frac{t_c E[\widetilde{c}_2]}{1+r} - t_I I - \overline{Q} \geq 0 \tag{17}$$

Note that $\overline{s}(1 + t_c) = s$ and $\overline{I}(1 + t_c) = I$. Thus, the government budget constraint can be rewritten as

$$t_w wl + \frac{t_{ks} r\overline{s}(1+t_c) + t_{kr}\mu \overline{I}(1+t_c) - D}{1+r} + t_c c_1 + \frac{t_c E[\widetilde{c}_2]}{1+r} - t_I \overline{I}(1+t_c) - \overline{Q} \geq 0. \tag{18}$$

Thus, consider the following constrained maximization problem:

$$W(t_c) \equiv \max_{\{t_{ks}, t_{kr}, t_I, D, t_w\}} V(\overline{w}, \overline{D}, t_{ks}, t_{kr}; t_c) \tag{19}$$

$$\text{s.t. } t_w wl^* + \frac{t_{ks} r\overline{s}^*(1+t_c) + t_{kr}\mu \overline{I}^*(1+t_c) - D}{1+r} + t_c c_1^* + \frac{t_c E[\widetilde{c}_2^*]}{1+r} - t_I \overline{I}^*(1+t_c) - \overline{Q} \geq 0$$

$$\overline{w} = w(1-t_w)/(1+t_c) \text{ and } \overline{D} = D/(1+t_c)$$

$$t_c \text{ is given}$$

Note that $W(t_c)$ is the maximized utility for a given level of the consumption tax when other taxes are chosen optimally. Let λ be the Lagrangian multiplier of the government budget constraint and let L be the associated Lagrangian function. Then, we can calculate the first order condition with respect to t_{ks}, t_{ki}, t_w and t_I for a given level of t_c. To save the space, the detailed first order conditions are calculated in Appendix. Our question is to calculate dW/dt_c and evaluate at $t_c = 0$. From the envelope theorem, $dW/dt_c = \partial L/\partial t_c$. Thus, dW/dt_c is

$$dW/dt_c|_{t_c=0} = \frac{\partial \overline{w}}{\partial t_c}\frac{\partial L}{\partial \overline{w}} + \frac{\partial \overline{D}}{\partial t_c}\frac{\partial L}{\partial \overline{D}} + \frac{\partial V}{\partial t_c} + \lambda\{\frac{t_{ks} r \overline{s}^*}{1+r} + \frac{t_{kr}\mu \overline{I}^*}{1+r} + c_1^* + \frac{E[\widetilde{c}_2^*]}{1+r} - t_I \overline{I}^*\} \tag{20}$$

Applying the first order conditions of t_w and D and $\partial V/\partial t_c = 0$:

$$dW/dt_c|_{t_c=0} = -\lambda w(1-t_w)l^* + \{-D\}\{\lambda\frac{1}{1+r}\} + \lambda\left\{\frac{t_{ks} r \overline{s}^*}{1+r} + \frac{t_{kr}\mu(e^*)\overline{I}^*}{1+r} + w(1-t_w)l^* - \overline{s}^* - (1-t_I)\overline{I}^* + \frac{[1+r(1-t_{ks})]\overline{s}^* + [1+\mu(e^*)(1-t_{kr})]\overline{I}^* + D}{1+r} - t_I\overline{I}^*\right\} \tag{21}$$

$$= \lambda\left\{\frac{\mu(e^*) - r}{1+r}\right\}\overline{I}^* \tag{22}$$

This result shows that introducing the consumption tax will improve welfare even when the government applies a linear capital income tax to absorb idiosyncratic risks as long as the expected rate of return from risky assets is greater than the rate of return from the safe asset.[6]

[6]Reader might think that the presence of constant part of the capital income tax implies that the government can use a lump-sum tax and, as a result, optimal consumption tax should be zero. However, such intuition does not hold in our model. Because the market is incomplete, the availability of the lump-sum tax does not imply that other distortionary tax should be zero.

2.3 Risk sharing in the presence of the nonlinear capital income tax

The above-described results showed that, even if the government institutes a linear capital income tax for the purpose of risk-sharing, it is still efficient to introduce a consumption tax when the market is incomplete. However, it could be argued that the power of a linear capital income tax to share in those risks is limited and that if the government instead uses a more flexible, non-linear capital income tax the result will be different.

Here, it is assumed that the government imposes a tax on the net return from risky investments and that the tax liability is a non-linear function of the net return from those investments. The ability of the consumption tax to increase welfare under such circumstances is then determined.

Let z be the realized value of the random value of $\widetilde{r}$. The non-linear capital income tax is a function of $R \equiv zI$. Let $T(R)$ be this non-linear capital income tax function. Then, for a given $T(R), t_{ks}, t_w$ and t_c , an individual will solve the following maximization problem:

$$\max_{\{l,s,I,e\}} u(\frac{wl(1-t_w)}{1+t_c} - \frac{s}{1+t_c} - \frac{(1-t_I)I}{1+t_c}) - h(l) - v(e) + \frac{1}{1+\rho} E[u(\frac{(1+r(1-t_{ks}))s + (1+\widetilde{r}I) - T(\widetilde{r}I)}{1+t_c})] \tag{23}$$

The government budget constraint is

$$t_w wl + \frac{t_{ks} rs + E[T(\widetilde{r}I)]}{1+r} + t_c c_1 + \frac{t_c E[\widetilde{c}_2]}{1+r} - t_I I - \overline{Q} \geq 0 \tag{24}$$

The government sets $T(R), t_{ks}, t_w$ and t_I to maximize the indirect utility determined by equation (23) subject to the government budget constraint (24) for a given value of t_c.

The concern here is to examine whether increasing t_c from zero will increase the indirect utility. To conduct such a thought experiment, the method first developed by Christiansen (1984) is applied. Let $T(R;\eta)$ be a non-linear tax function in which η is a parameter that shifts the tax schedule. Also, let $T^*(R)$ be the optimal nonlinear tax schedule that maximizes (23) subject to the government budget constraint (24) when $t_c = 0$. It is assumed that when $\eta = 0$ the nonlinear tax function $T(R;\eta)$ becomes $T^*(R)$. More specifically, $T(R;\eta)$ is defined as follows:

$$T(R;\eta) = T^*(R) + \eta \times G(R) \tag{25}$$

where $G(R)$ is an arbitrary function. Accordingly, a representative agent will solve the following optimization problem given t_c, t_w, t_I and $T(R;\eta)$:

$$\begin{aligned}
&\max_{\{l,s,I,e\}} u(\frac{wl(1-t_w)}{1+t_c} - \frac{s}{1+t_c} - \frac{(1-t_I)I}{1+t_c}) - h(l) - v(e) \\
&+ \frac{1}{1+\rho} E[u(\frac{(1+r(1-t_{ks}))s + (1+\widetilde{r})I - T(\widetilde{r}I;\eta)}{1+t_c})].
\end{aligned} \tag{26}$$

The objective of the government is to maximize the above indirect utility function subject to the government budget constraint. The purpose here is to examine whether an introduction of the consumption tax increases welfare. To consider this issue, it is useful to analyze the problem through several steps. Accordingly, $\overline{w}$, $\overline{s}$ and $\overline{I}$ are defined in the first period as $\overline{w} \equiv w(1-t_w)/(1+t_c)$, $\overline{s} \equiv \frac{s}{1+t_c}$, $\overline{I} \equiv \frac{I}{1+t_c}$ and the following sub-indirect utility function is considered:

$$\Psi(\overline{w}, t_I, \overline{s}, \overline{I}) = \max_{\{l\}} u(\overline{w}l - \overline{s} - (1-t_I)\overline{I}) - h(l) \tag{27}$$

Note that this sub-indirect utility function is concave with respect to $\overline{s}$ and $\overline{I}$ as long as the consumption good is a normal good. Also, note that l is a function of $\overline{w}$,t_I,$\overline{s}$ and $\overline{I}$. Given this sub-indirect utility function, the following indirect utility function can be defined:

$$V(\overline{w}, t_I, t_{ks}, \eta; t_c) \equiv \max_{\{\overline{s}, \overline{I}, e\}} \Psi(\overline{w}, t_I, \overline{s}, \overline{I}) - v(e) + \frac{1}{1+\rho} E[u(1 + r(1 - t_{ks}))\overline{s} + (1 + \widetilde{r})\overline{I} - \frac{T(\widetilde{r}\overline{I}(1 + t_c); \eta)}{1 + t_c})] \tag{28}$$

In the above optimization problem, $\overline{s}$,$\overline{I}$and e can be considered as functions of $t_{ks}, \eta, t_I$and t_c. Given this indirect utility function, the following constrained optimization problem for given t_c can be solved:

$$W(t_c) \equiv \max_{\{t_w, t_I, \eta, t_{ks}\}} V(\overline{w}, t_I, t_{ks}, \eta; t_c,) + \lambda \left\{ wlt_w + t_c c_1 + \frac{t_c E[\widetilde{c}_2]}{1+r} + \frac{r t_{ks} \overline{s}(1 + t_c)}{1+r} + E[\frac{T(\widetilde{r}\overline{I}(1 + t_c); \eta)}{1+r}] - t_I \overline{I}(1 + t_c) - \overline{Q} \right\} \tag{29}$$

where λ is the Lagrangian multiplier of the government budget constraint.

$W(t_c)$ is the indirect utility when the government chooses an optimal wage tax rate, a capital income tax rate for the safe asset and a nonlinear capital income tax schedule for a given consumption tax rate t_c. Now, consider the effect of increasing the consumption tax rate from zero. From the envelope theorem and the first order conditions defined above (see Appendix):

$$\left.\frac{dW}{dt_c}\right|_{t_c=0} = \lambda\{\frac{\mu(e) - r}{1+r}\}\overline{I} \tag{30}$$

The result can be summarized as follows:

Proposition

Increasing the consumption tax rate, t_c, from zero in the presence of a optimal wage tax, investment subsidy, nonlinear or linear capital income tax is welfare improving as long as

the expected rate of return from the risky investment is higher than the rate of return from the safe asset at t_c=0

Economic interpretation of (30) is as follows. When the government increases the consumption rate by $\triangle\ t_c$ from zero, the investment on the risky asset and the safe asset increases by $\triangle\ t_c$ in order to pay the consumption tax in the second period. Thus, in absolute terms, the amount of investment in risky asset increases by the initial amount of investment on risky investment. For each dollar of risky investment, since the expected rate of return from risky investment is higher than that of the safe asset and there is no macroeconomic uncertainty. Thus, there is welfare gain by $\{\mu(e) - r\}\overline{I} \times \triangle\ t_c$. In the present value, this is equal to $\{\mu(e) - r\}\overline{I} \times \triangle\ t_c/(1 + r)$.

2.4 Intuition and Discussion

At this point, readers are likely to have several questions. The first is "Why does using the consumption tax together with the wage tax increase efficiency even if a non-linear capital income tax has already been instituted to absorb idiosyncratic risks?" Based on the assumption that a non-linear capital income tax can be applied very flexibly, one might wonder why a consumption tax still plays a role.

The basic intuition that a consumption tax increases efficiency comes from the interaction of the risk-sharing effect, which is already discussed in this paper, and its timing effect, which was first discussed by Summers (1981). In that paper, it was argued that the wage tax and the consumption tax have different timing effects and, as a result, different effects on individual savings. As in many life-cycle models, in our model a switch from the wage tax to the consumption tax shifts some of the tax burden from the first (early) period to the second (late) period. This implies that consumers need to increase their investments in both safe assets and risky assets in order to be able to pay more taxes in the second period, when the government switches from the wage tax to the consumption tax. However, the expected rate of return from risky assets is higher than that from safe assets due to the assumption of an incomplete capital market, although there is no macroeconomic risk for risky assets. Thus, from a social-planning perspective, it is desirable to increase investments in risky assets while simultaneously sharing those risks. The shift from the wage tax to the consumption tax not only makes risk-sharing possible, it also compels individuals to increase investments in both risky assets and safe assets due to the timing effect of the consumption tax. The effect of this shift does not disappear even in the presence of a non-linear capital income tax.

Certainly, a non-linear capital income tax and investment subsidies can be used to increase investments in risky assets and to enhance risk-sharing. However, those policy instruments also introduce distortions. By contrast, a consumption tax can increase investments into risky assets and improve risk-sharing without disincentive effects on investments because the consumption tax does not change the relative price of consumption during different periods. Although the consumption tax has limited risk-sharing power, the marginal cost of introducing this tax is zero. Thus, even if the government applies a non-linear capital income tax to allow for risk-sharing, introduction of a consumption tax in the presence of the wage tax improves public welfare.

The second question is the degree of combination of the two taxes. In the model discussed here, as the logic of the analysis shows, a complete switch from the wage tax to the consumption tax is optimal. Thus, one might argue that the analysis does not explain an optimal mix of the consumption tax and the wage tax. However, this shortcoming can be easily overcome by introducing ex-ante heterogeneity. When the wage tax and the consumption tax are compared, one of the advantages of the wage tax is that it can be used to redistribute income. Although, theoretically, the consumption tax can also be used for income redistribution through the expenditure tax, administration of the latter is quite difficult. Thus, if heterogeneity and the necessity of income redistribution are introduced ex-ante, a combination of the wage tax and the consumption tax becomes optimal. In that situation, the role of the wage tax is limited to income redistribution while the consumption tax, together with capital income tax, plays a role in raising revenue and risk-sharing.

3 Implication and Conclusions

The present study has shown that an introduction of the consumption tax in the presence of the wage tax increases economic efficiency when a capital market is incomplete, even if the government has instituted either a linear or a non-linear capital income tax. The improved efficiency comes from the fact that the consumption tax will shift the tax burden in later periods and, due to the timing effect of the consumption tax (Summers 1981), increases investments in risky assets. The rate of return on risky investments being higher than safe assets, but with no macroeconomic risk on risky investments, increased investment on risky assets increases economic efficiency.

This intuition has an important implication for the privatization of public pension. In the standard theory, it has been assumed that the expected rates of returns of the government fund and the individual fund are the same. As a result, it is argued that whether the fund is managed by the government or individual does not matter and that the privatization of the public pension does not bring the real welfare gain. However, when a capital market is incomplete, individuals might be facing a higher expected rate of return, with higher variance of the idiosyncratic rate of return due to incompleteness of capital market(but no macroeconomic risk), than that the government faces. In such a case, the government might be able to improve welfare by privatizing social security, let individuals invest in risky assets with a nonlinear tax schedule on the return from those investment to absorb idyosyncratic risks.

4 Appendix

4.1 First order conditions under a linear capital income tax

The first order conditions are:

$$t_w : \frac{\partial \overline{w}}{\partial t_w}\frac{\partial L}{\partial \overline{w}} + \lambda w l^* = 0$$

$$D : \frac{\partial \overline{D}}{\partial D}\frac{\partial L}{\partial \overline{D}} - \lambda \frac{1}{1+r} = 0$$

$$t_{ks} : \frac{\partial V}{\partial t_{ks}} + \lambda w \frac{\partial l^*}{\partial t_{ks}} + \lambda \frac{t_{ks} r(1+t_c)}{1+r}\frac{\partial \overline{s}^*}{\partial t_{ks}} + \lambda \frac{t_{kr}\mu(1+t_c)}{1+r}\frac{\partial \overline{I}^*}{\partial t_{ks}}$$

$$+\lambda t_c \frac{\partial c_1^*}{\partial t_{ks}} + \lambda \frac{t_c}{1+r}\frac{\partial E[\widetilde{c}_2^*]}{\partial t_{ks}} - \lambda t_I(1+t_c)\frac{\partial \overline{I}^*}{\partial t_{ks}} + \lambda \frac{r\overline{s}^*(1+t_c)}{1+r} = 0$$

$$t_{kr} : \frac{\partial V}{\partial t_{kr}} + \lambda w \frac{\partial l^*}{\partial t_{kr}} + \lambda \frac{t_{ks} r(1+t_c)}{1+r}\frac{\partial \overline{s}^*}{\partial t_{ks}} + \lambda \frac{t_{kr}\mu(1+t_c)}{1+r}\frac{\partial \overline{I}^*}{\partial t_{ks}}$$

$$+\lambda t_c \frac{\partial c_1^*}{\partial t_{ks}} + \lambda \frac{t_c}{1+r}\frac{\partial E[\widetilde{c}_2^*]}{\partial t_{kr}} - \lambda t_I(1+t_c)\frac{\partial \overline{I}^*}{\partial t_{kr}} + \lambda \frac{\mu\overline{I}^*(1+t_c)}{1+r} = 0$$

$$t_I : \frac{\partial V}{\partial t_I} + \lambda w \frac{\partial l^*}{\partial t_I} + \lambda \frac{t_{ks} r(1+t_c)}{1+r}\frac{\partial \overline{s}^*}{\partial t_I} + \lambda \frac{t_{kr}\mu(1+t_c)}{1+r}\frac{\partial \overline{I}^*}{\partial t_I} + \lambda t_c \frac{\partial c_1^*}{\partial t_I}$$

$$+\lambda \frac{t_c}{1+r}\frac{\partial E[\widetilde{c}_2^*]}{\partial t_I} - \lambda t_I(1+t_c)\frac{\partial \overline{I}^*}{\partial t_I} - \lambda \overline{I}^*(1+t_c) = 0$$

4.2 The effect of increasing consumption tax under a non-linear capital

income tax

The associated Lagrangian function is:

$$
\begin{aligned}
L = \ & V(\overline{w}, t_I, t_{ks}, \eta; t_c,) \\
& + \lambda \left\{ wlt_w + t_c c_1 + \frac{t_c E[\widetilde{c}_2]}{1+r} + \frac{rt_{ks}\overline{s}(1+t_c)}{1+r} + E[\frac{T(\widetilde{r}\overline{I}(1+t_c);\eta)}{1+r}] - t_I \overline{I}(1+t_c) - \overline{Q} \right\}
\end{aligned} \tag{31}
$$

The first order conditions regarding t_w, t_{ks}, t_I and η, are:

$$
t_w : \frac{\partial \overline{w}}{\partial t_w}\frac{\partial L}{\partial \overline{w}} + \lambda wl = 0 \tag{32}
$$

$$
\begin{aligned}
t_{ks} : -\frac{1}{1+\rho}E[u'(\widetilde{c}_2)r\overline{s}] + \lambda \Bigg\{ & wt_w \frac{\partial l}{\partial \overline{s}}\frac{\partial \overline{s}}{\partial t_{ks}} + wt_w \frac{\partial l}{\partial \overline{I}}\frac{\partial \overline{I}}{\partial t_{ks}} + t_c \frac{\partial c_1}{\partial t_{ks}} + \frac{t_c}{1+r}\frac{\partial E[\widetilde{c}_2]}{\partial t_{ks}} \\
& \frac{r\overline{s}(1+t_c)}{1+r} + \frac{rt_{ks}}{1+r}\frac{(1+t_c)\partial \overline{s}}{\partial t_{ks}} + \left(E[\frac{T'(\widetilde{r}\overline{I}(1+t_c);\eta)\widetilde{r}}{1+r}] - t_I \right) \frac{\partial \overline{I}(1+t_c)}{\partial t_{ks}} \\
& + E[T'(\widetilde{r}\overline{I}(1+t_c);\eta)\overline{I}(1+t_c)\frac{\partial \widetilde{r}}{\partial e}\frac{\partial e}{\partial t_{ks}}] \Bigg\} = 0
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
t_I : \frac{\partial \Psi}{\partial t_I} + \lambda \Bigg\{ & wt_w \frac{\partial l}{\partial t_I} + wt_w \frac{\partial l}{\partial \overline{s}}\frac{\partial \overline{s}}{\partial t_I} + wt_w \frac{\partial l}{\partial \overline{I}}\frac{\partial \overline{I}}{\partial t_I} + t_c \frac{\partial c_1}{\partial t_I} + \frac{t_c}{1+r}\frac{\partial E[\widetilde{c}_2]}{\partial t_I} \\
& + \frac{rt_{ks}}{1+r}\frac{(1+t_c)\partial \overline{s}}{\partial t_I} + \left(E[\frac{T'(\widetilde{r}\overline{I}(1+t_c);\eta)\widetilde{r}}{1+r}] - t_I \right) \frac{\partial \overline{I}(1+t_c)}{\partial t_I} \\
& + E[T'(\widetilde{r}\overline{I}(1+t_c);\eta)\overline{I}(1+t_c)\frac{\partial \widetilde{r}}{\partial e}\frac{\partial e}{\partial t_I}] \Bigg\} = 0
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
& \eta : -\frac{1}{1+\rho}E[u'(c_2)\frac{1}{1+t_c}\frac{\partial T}{\partial \eta}] + \lambda \Bigg\{ \frac{\partial \overline{s}}{\partial \eta}\{wt_w \frac{\partial l}{\partial \overline{s}} + \frac{rt_{ks}}{1+r}\} \\
& + \frac{\partial \overline{I}}{\partial \eta}\{wt_w \frac{\partial l}{\partial \overline{I}} + E[T'(\widetilde{r}I(1+t_c);\eta)(1+t_c)\widetilde{r}] - (1+t_c)t_I\} + E[T'(\widetilde{r}I(1+t_c);\eta)\overline{I}(1+t_c)\frac{\partial \widetilde{r}}{\partial e}\frac{\partial e}{\partial \eta}] \\
& + E[\frac{1}{1+r}\frac{\partial T}{\partial \eta}\frac{1}{1+t_c}] \Bigg\} = 0
\end{aligned} \tag{35}
$$

The effect of increasing the consumption tax from zero is equal to:

$$\begin{aligned}
\left.\frac{dW}{dt_c}\right|_{t_c=0} &= \frac{1}{1+\rho}E[u'(c_2)\{T - T'\widetilde{r}\overline{I}\}\} \\
&+ \frac{\partial \overline{w}}{\partial t_c}\frac{\partial L}{\partial \overline{w}} \\
&+ \frac{\partial \overline{s}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{s}} + \frac{r t_{ks}}{1+r}\} + \frac{\partial \overline{I}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{I}} + \lambda E[\frac{T'(\widetilde{r}I;\eta)\widetilde{r}}{1+r}] - t_I\} \\
&+ \frac{\partial e}{\partial t_c}\{\lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\overline{I}}{1+r}\frac{\partial \widetilde{r}}{\partial e}] \\
&+ \lambda\{c_1 + \frac{E[c_2]}{1+r} + \frac{r t_{ks}\overline{s}}{1+r} + E[\frac{T(\widetilde{r}\overline{I};\eta)\widetilde{r}\overline{I}}{1+r}] - t_I\overline{I}\} \qquad (36) \\
&= \frac{1}{1+\rho}E[u'(c_2)\{T - T'\widetilde{r}\overline{I}\}] \\
&- w(1-t_w)\frac{\partial L}{\partial \overline{w}} + \frac{\partial \overline{s}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{s}} + \frac{r t_{ks}}{1+r}\} + \frac{\partial \overline{I}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{I}} + \lambda E[\frac{T'(\widetilde{r}I;\eta)\widetilde{r}}{1+r}] - t_I\} \\
&+ \frac{\partial e}{\partial t_c}\lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\overline{I}}{1+r}\frac{\partial \widetilde{r}}{\partial e}] + \lambda\{c_1 + \frac{E[c_2]}{1+r} + \frac{r t_{ks}\overline{s}}{1+r} + E[\frac{T(\widetilde{r}\overline{I};\eta)\widetilde{r}\overline{I}}{1+r}] - t_I\overline{I}\} \qquad (37)
\end{aligned}$$

By using the first order condition of t_w, we have

$$\begin{aligned}
\left.\frac{dW}{dt_c}\right|_{t_c=0} &= \frac{1}{1+\rho}E[u'(c_2)\{T - T'r\overline{I}\}] \\
&- \lambda w(1-t_w)l \\
&+ \frac{\partial \overline{s}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{s}} + \frac{r t_{ks}}{1+r}\} + \frac{\partial \overline{I}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{I}} + \lambda E[\frac{T'(\widetilde{r}I;\eta)\widetilde{r}}{1+r}] - t_I\} \\
&+ \frac{\partial e}{\partial t_c}\lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\overline{I}}{1+r}\frac{\partial \widetilde{r}}{\partial e}] + \lambda\{c_1 + \frac{E[c_2]}{1+r} + \frac{r t_{ks}\overline{s}}{1+r} + E[\frac{T'(\widetilde{r}\overline{I};\eta)\widetilde{r}\overline{I}}{1+r}] - t_I\overline{I}\} \qquad (38)
\end{aligned}$$

Note that $c_1 = w(1-t_w)l - \overline{s} - (1-t_I)\overline{I}$ and $E[c_2] = (1 + r(1-t_{ks}))\overline{s} + (1+\mu)\overline{I} - E[T(\widetilde{r}I;\eta)]$

at $t_c = 0$. Thus, we have

$$\left.\frac{dW}{dt_c}\right|_{t_c=0} = \frac{1}{1+\rho}E[u'(c_2)\{T - T'\widetilde{r}\overline{I}\}]$$
$$+ \frac{\partial \overline{s}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{s}} + \frac{r t_{ks}}{1+r}\} + \frac{\partial \overline{I}}{\partial t_c}\{\lambda w t_w \frac{\partial l}{\partial \overline{I}} + \lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\widetilde{r}}{1+r}] - t_I\}$$
$$+ \frac{\partial e}{\partial t_c}\lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\overline{I}}{1+r}\frac{\partial \widetilde{r}}{\partial e}]$$
$$+ \lambda\left\{\{\frac{\mu - r}{1+r}\}\overline{I} + E[\frac{T'(\widetilde{r}\overline{I};\eta)\widetilde{r}\overline{I}}{1+r}] - E[\frac{T(\widetilde{r}\overline{I};\eta)}{1+r}]\right\} \tag{39}$$

At this point, we need to use the information on the first order condition on η. For this purpose, assume that the perturbation term $G(R) = T^*(R) - T^{*\prime}(R) \times R$ and that $T(R;\eta) = T^*(R) + \eta \times G(R) = T^*(R) + \eta[\,T^*(R) - T^{*\prime}(R) \times R]$. This perturbation term has a special economic meaning. This perturbation term is the increase of the tax burden due to the consumption tax. When $G(R)$ is defined in this way, we have $\frac{\partial T}{\partial \eta} = T^*(R) - T^{*\prime} \times R$. From the first order condition of η , we have

$$\frac{1}{1+\rho}E[u'(c_2)\{T^*(R) - T^{*\prime} \times R\}] = \lambda\left\{\frac{\partial \overline{s}}{\partial \eta}\{w t_w \frac{\partial l}{\partial \overline{s}} + \frac{r t_{ks}}{1+r}\}\right.$$
$$+\frac{\partial \overline{I}}{\partial \eta}\{w t_w \frac{\partial l}{\partial \overline{I}} + E[T'(\widetilde{r}I;\eta)\widetilde{r}] - t_I\} + E[T'(\widetilde{r}I;\eta)\overline{I}\frac{\partial \widetilde{r}}{\partial e}\frac{\partial e}{\partial \eta}]$$
$$\left.+E[\frac{1}{1+r}\{T^*(R) - T^{*\prime} \times R\}]\right\} \tag{40}$$

Note that by the definition, $R = \widetilde{r}\overline{I}.$ Therefore, we have

$$\left.\frac{dW}{dt_c}\right|_{t_c=0} = \{\frac{\partial \overline{s}}{\partial t_c} + \frac{\partial \overline{s}}{\partial \eta}\}\{\lambda w t_w \frac{\partial l}{\partial \overline{s}} + \frac{r t_{ks}}{1+r}\} + \{\frac{\partial \overline{I}}{\partial t_c} + \frac{\partial \overline{I}}{\partial \eta}\}\{\lambda w t_w \frac{\partial l}{\partial \overline{I}} + \lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\widetilde{r}}{1+r}] - t_I\}$$
$$+\{\frac{\partial e}{\partial t_c} + \frac{\partial e}{\partial \eta}\}\{\lambda E[\frac{T'(\widetilde{r}\overline{I};\eta)\overline{I}}{1+r}\frac{\partial \widetilde{r}}{\partial e}] + \lambda\{\frac{\mu - r}{1+r}\}\overline{I} \tag{41}$$

Now we need to calculate $\frac{\partial \overline{s}}{\partial t_c}$, $\frac{\partial \overline{I}}{\partial t_c}\frac{\partial e}{\partial t_c}$, $\frac{\partial \overline{s}}{\partial \eta}$, $\frac{\partial \overline{I}}{\partial \eta}$ and $\frac{\partial e}{\partial \eta}$. The first order conditions of consumers for $\overline{s}$, $\overline{I}$ and e for given t_c are

$$\frac{\partial \Psi}{\partial \overline{s}} + \frac{1}{1+\rho}E[u'(c_2)\{1+r(1-t_{ks})\}] = 0$$
$$\frac{\partial \Psi}{\partial \overline{I}} + \frac{1}{1+\rho}E[u'(c_2)\{1+\widetilde{r}-T'\widetilde{r}\}] = 0$$
$$-v'(e) + \frac{1}{1+\rho}E[u'(c_2)\frac{\partial \widetilde{r}}{\partial e}\{\overline{I}-T'\overline{I}\}] = 0$$

Now, let SOC be the matrix that is derived from the second order condition. From the concavity of the objective function with respect to $\overline{s}$, $\overline{I}$ and e, the determinant of SOC is strictly negative. Then, $\frac{\partial \overline{s}}{\partial t_c}$, $\frac{\partial \overline{I}}{\partial t_c}$ and $\frac{\partial e}{\partial t_c}$ are the solution of the following equations.

$$SOC \times \begin{bmatrix} \frac{\partial \overline{s}}{\partial t_c} \\ \frac{\partial \overline{I}}{\partial t_c} \\ \frac{\partial e}{\partial t_c} \end{bmatrix} + \begin{bmatrix} \frac{1}{1+\rho}E[u''(c_2)\{1+r(1-t_{ks})\}\{T-T'\widetilde{r}\overline{I}\}] \\ \frac{1}{1+\rho}E[u''(c_2)\{1+\widetilde{r}-T'\widetilde{r}\}\{T-T'\widetilde{r}\overline{I}\} - E[u'(c_2)T''\widetilde{r}^2\overline{I}] \\ \frac{1}{1+\rho}E[u''(c_2)\frac{\partial \widetilde{r}}{\partial e}\{\overline{I}-T'\overline{I}\}\{T-T'\widetilde{r}\overline{I}\} - E[u'(c_2)\frac{\partial \widetilde{r}}{\partial e}T''\widetilde{r}\overline{I}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (42)$$

As for $\frac{\partial \overline{s}}{\partial \eta}$, $\frac{\partial \overline{I}}{\partial \eta}$ and $\frac{\partial e}{\partial \eta}$, we have

$$SOC \times \begin{bmatrix} \frac{\partial \overline{s}}{\partial \eta} \\ \frac{\partial \overline{I}}{\partial \eta} \\ \frac{\partial e}{\partial \eta} \end{bmatrix} + \begin{bmatrix} -\frac{1}{1+\rho}E[u''(c_2)\{1+r(1-t_{ks})\}T_\eta] \\ -\frac{1}{1+\rho}E[u''(c_2)\{1+\widetilde{r}-T'\widetilde{r}\}T_\eta] - E[u'(c_2)\frac{\partial T'}{\partial \eta}\widetilde{r}] \\ -\frac{1}{1+\rho}E[u''(c_2)\frac{\partial \widetilde{r}}{\partial e}\{\overline{I}-T'\overline{I}\}T_\eta - E[u'(c_2)\frac{\partial T'}{\partial \eta}\frac{\partial \widetilde{r}}{\partial e}\overline{I}] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Note that $T_\eta = T - T'\widetilde{r}\overline{I}$ and that $\frac{\partial T'}{\partial \eta} = \frac{\partial [T_\eta]}{\partial R} = -T''R$. Therefore,

$$SOC \times \begin{bmatrix} \frac{\partial \overline{s}}{\partial \eta} \\ \frac{\partial \overline{I}}{\partial \eta} \\ \frac{\partial e}{\partial \eta} \end{bmatrix} + \begin{bmatrix} -\frac{1}{1+\rho}E[u''(c_2)\{1+r(1-t_{ks})\}\{T-T'\widetilde{r}\overline{I}\}] \\ -\frac{1}{1+\rho}E[u''(c_2)\{\widetilde{r}-T'\widetilde{r}\}\{T-T'\widetilde{r}\overline{I}\} + [u'(c_2)T''\widetilde{r}^2\overline{I}] \\ -\frac{1}{1+\rho}E[u''(c_2)\frac{\partial \widetilde{r}}{\partial e}\{\overline{I}-T'\overline{I}\}\{T-T'\widetilde{r}\overline{I}\} + E[u'(c_2)\frac{\partial \widetilde{r}}{\partial e}T''\widetilde{r}\overline{I}^2] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (43)$$

From (42) and (43), we have $\frac{\partial \overline{s}}{\partial t_c} = -\frac{\partial \overline{s}}{\partial \eta}$,$\frac{\partial \overline{I}}{\partial t_c} = -\frac{\partial \overline{I}}{\partial \eta}$ and $\frac{\partial e}{\partial t_c} = -\frac{\partial e}{\partial \eta}$ Thus, dW/dt_c is

$$\left.\frac{dW}{dt_c}\right|_{t_c=0} = \lambda\{\frac{\mu(e)-r}{1+r}\}\overline{I}$$

From the assumption, we have $\mu(e) > r$. Therefore, introducing the consumption tax is welfare-improving.

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