

**Tsukuba Economics Working Papers**  
**No. 2013-004**

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by

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August 2013

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# Pareto-improving Immigration and Its Effect on Capital Accumulation in the Presence of Social Security

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August 5, 2013

## Abstract

The effect of accepting more immigrants on welfare in the presence of a pay-as-you-go social security system is analyzed qualitatively and quantitatively. First, it is shown that if initially there exist intergenerational government transfers from the young to the old, the government can lead an economy to the (modified) golden rule level within a finite time in a Pareto-improving way by increasing the percentage of immigrants to natives (PITN). Second, using the computational overlapping generation model, the welfare gain is calculated of increasing the PITN from 15.5 percent to 25.5 percent and years needed to reach the (modified) golden rule level in a Pareto-improving way in a model economy. The simulation shows that the present value of the welfare gain of increasing the PITN comprises 23 percent of the initial GDP. It takes 112 years for the model economy to reach the golden rule level in a Pareto-improving way.

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<sup>†</sup>I appreciate comments from the participants of the Canadian Public Economics Meeting(Kingston), the North American Summer Econometric Society Meeting (St. Louis), the European Econometrics Society Meeting (Oslo), Macroeconomics workshop at the University of Tokyo. I appreciate comments from Kosuke Aoki, Masayoshi Hayashi and Shinichi Nishiyama. I especially appreciate comments from Pierre Pestieau for pointing out the literature of the Serendipity theorem.

# 1 Introduction

Transforming a pay-as-you-go(PYGO) social security system into a funded system is not easy. When the PYGO social security system is changed to a funded system, some generations must bear the so called “the double burden”, such that a young generation needs to pay the social security tax twice. Thus, although the transition from a PYGO social security to a funded system is desirable since a PYGO social security system causes under-accumulation of capital, it is difficult to transit in a Pareto-improving way.

On the other hand, in the face of the fiscal problems caused by negative demographic shocks in the presence of PYGO social security, policy makers have recently shown increased interests in accepting more immigrants. For example, the IMF report on the Japanese economy (2012), which is suffering as a result of an extremely low fertility rate and practical restrictions on inflow of immigrants, states that facilitating a modest level of immigration could have a large payoff for the Japanese economy. However, although using immigrants to solve the fiscal problem of PYGO social security in the short run is attractive, its desirability in the long run is unclear. Increasing the percentage of immigrants to natives implies a higher population growth rate in a host country given the higher fertility rate of immigrants. With a neoclassical production function that exhibits the diminishing marginal product of capital, the standard growth model (Ramsey (1928) and Solow (1964)) predicts that such a higher population growth rate leads to both lower level of capital stock per capita and income per capita, starting from a dynamically efficient initial steady state. Thus, it is not clear how such an increase of immigrants can help to solve the problem of PYGO social security in the long run. The literature also presents an unclear picture. In the literature on public finance, there is increasing interest in the effect on the social welfare of accepting more immigrants. By using the computational overlapping generation model (Auerbach and Kotlikoff model(Auerbach and Kotlikoff (1998)), Storesletten (2000) argues that accepting a particular type of immigrants, (skilled who are of an age such that they will not be able to claim the social security benefit because they will not be able to satisfy the minimum requirement of the duration of social security tax payments) will increase the social welfare in the presence of the retirement of the baby boom generation. In contrast, Fehr, Jokisch and Kotlikoff (2004) argue that there will be no such a welfare gain. Feldstein (2006) analyzes the effect of immigration in Spain and concludes that immigration does not bring a welfare gain while Collado, Iturbe-Ormaetxeand and Valera (2004) argue that accepting more immigrants brings a positive welfare gain to Spain.

Given the mixed results in the literature regarding the effect of accepting more immigrants on the social welfare and the theoretical prediction of the standard growth model, a natural question arises whether or not, from a theoretical standpoint, accepting more immigrants Pareto-improves welfare in the presence of PYGO social security. With a neoclassical production function that exhibits diminishing marginal product of capital, a decrease of income per capita at the steady state seems inevitable when the economy experiences a higher population growth rate due to an increased inflow of immigrants. Yet, if this is so, one may wonder why the previous studies have arrived at such different results regarding the effect on welfare of accepting more immigrants.

Motivated by those questions, I analyze theoretically and qualitatively the effect on welfare of accepting more immigrants and increasing the population growth rate. I conclude that accepting more immigrants and increasing the population growth rate Pareto-improves welfare and, to a large extent, solves in a Pareto-improving way the problem of under-accumulation of capital that is caused by implementing a PYGO social security system.<sup>1</sup> More specifically, firstly, using the overlapping generation model developed by Diamond (1965), I show that in an economy with or without distorting taxes it is Pareto-improving to increase the percentage of immigrants to natives (PITN) if there exist intergenerational transfers, in the sense that the marginal product of labor of a young individual is greater than the amount of resources that she receives while she is young, which is equal to the sum of the after-tax income when she is young and the amount of publicly provided private goods for the young (the MPL condition). Note that the amount of resources an individual receives when she is young does not include the social security benefit that she receives when she reaches retirement age. Thus, in the presence of PYGO social security, this MPL condition is likely to be satisfied. Secondly, I show analytically that when this MPL condition is satisfied, the government can lead the economy to the (modified) golden rule level in a Pareto-improving way within a finite time by putting in savings the government budget surplus, which is obtained by increasing the PITN. Note that when the economy reaches the golden rule level, the problem of under-accumulation of capital caused by the PYGO social security is solved for all practical purposes. Third, I quantify this Pareto-improving welfare gain that is yielded by increasing the PITN in the presence of a PYGO social security system and calculate the year needed to reach the (modified) golden rule

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<sup>1</sup>The Matlab code which is used for this simulation is available from the Journal' website and from the author.

level in a Pareto-improving way by using the computational overlapping generation model developed by Auerbach and Kotlikoff (1987). I consider a moderate increase of the PITN, such that the PITN starts to increase from 15.5 percent, reaches 25.5 percent at the 80th year and remains constant at 25.5 percent in later years.<sup>2</sup> With this speed of increase of the PITN and in the model that mimics important dimensions of the US economy, my simulation shows it takes a minimum of 112 years for the model economy to reach the golden rule level in a Pareto-improving way. On the new balanced growth path, the capital stock per efficient unit of labor increases by 102 percent and the publicly provided private goods per capita increases by 36 percent. When the target capital stock is set at the modified golden rule level with 3 percent of intergenerational discount rate, it takes 65 years to reach the modified golden rule level in a Pareto-improving way and the capital stock per efficient unit of labor increases by 18 percent. The present discounted value (PDV) of the Pareto-improved utility, measured by the expenditure function, of natives and their descendants, which does not include the increased utility of immigrants and their descendants, comprises 23 percent of the initial GDP. When the time to reach the target PITN is shortened to 42 years, the economy reaches the modified golden rule at the 59th year and the PDV of the Pareto improvement comprises 28 percent of the initial GDP. Finally, I conduct robustness checks by changing a number of parameter values, for example, the share of the surplus for the government savings, the replacement rate, the time preference rate, the risk aversion, the initial government debt(asset) level, the level of immigrants' earnings and the consumption of public services by immigrants. Those robustness checks show that the results of the simulation do not change substantially in magnitude for different parameter values. Both theoretical results and the computational results suggest the robustness of the welfare gain of increasing the PITN in the presence of PYGO social security.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents a theoretical analysis. Section 4 presents a simulation-based analysis using the computational overlapping generation model. Section 5 concludes.

## 2 Literature

In the theoretical literature on the effect of accepting immigrants, Razin (1999) is, to my best knowledge, the first paper that shows that accepting immigrants improves welfare.

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<sup>2</sup>15.5 percent initial PITN is obtained by using census 2000 data from the author's calculation. See section 4.2 for more detailed discussion.

He shows that in a small open economy model in which factor prices are fixed, accepting more immigrants can improve welfare in the presence of PYGO social security. However, in a subsequent paper, Razin and Sadka (2000) show that in an closed economy model in which capital accumulation and factor prices are endogenous, the result obtained under the assumption of a small open economy is not likely to hold. Subsequently, Razin, Sadka and Suwankiri (2011) analyzes a political economy model in which there is a conflict of interests among agents on accepting immigrants. They obtain results regarding on political behavior on immigration policy.<sup>3</sup>

In the empirical literature, there are three types of studies. The first generation of those studies conducts analysis with a short horizon which does not take into account the contribution of immigrants' children to the social security and tax revenue, and the cost burden which they impose to public services (Huddle(1993), Borjas(1994), Passel (1994), Simon (1984) and Akbari (1989)). The second generation of those studies conducts analysis with a long horizon (Generational Accounting) which takes into account the contribution of immigrants' children( and grand children, etc...) to the social security and tax revenue, and the cost burden that they impose to public services (Lee and Miller (1997), Auerbach and Preopoulous (1999), Storesletten (2003), and, Collado and Valera (2004)).<sup>4</sup>

The third generation of those studies conducts analysis with a dynamic general equilibrium model (Storesletten(1995, 2000), Canova and Ravn (2000), and Fehr, Joisch and Kotlikoff (2004)). Storesletten (1995) is the first paper to analyze the effect of accepting immigrants in a dynamic general equilibrium mode with aggregate uncertainty and Storesletten (2000) analyzes without aggregate uncertainty. He argues, in the US context, that by selecting a particular type of immigrants, the acceptance of immigrants can have a positive effect on the native population. Fehr, Joisch and Kotlikoff (2004) argue that there is no

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<sup>3</sup>In the theoretical literature, there is another strand that analyzes the optimal population growth in the growth model; this was started by Samuelson(1975). He analyzes the welfare maximizing optimal population growth rate in the standard neoclassical growth model. The subsequent analysis shows that the optimal population growth rate in many cases does not, in many cases, maximize the welfare; rather it minimizes welfare(Deardorff 1976). For discussion, see Michel and Pestieau(1999). Due to the concern of the non-optimality of the first order condition, I do not herein attempt to identify the optimal population growth rate or the optimal immigration rate. Rather, I check whether a small change in the population growth rate will Pareto-improve welfare or not.

<sup>4</sup>To my knowledge, Lee and Miller (1996) is the first paper that calculates the long-run fiscal impact of accepting immigrants. They calculate the tax revenue from and the cost to public services of, not only the current immigrants, but also the descendant of immigrants. They find that accepting immigrant bring a positive net present value to the government.

such welfare gain, or, even if there were to be a gain, it would be very small.<sup>5</sup>

This paper contributes to the existing literature in several ways. For the theoretical part, I show first that even in a closed economy in which capital accumulations and factor prices are endogenous, it is Pareto-improving to accept more immigrants in the presence of a PYGO social security system if the MPL condition is satisfied. This is contrary to the findings of Razin and Sadka (2000), but is consistent with the results of Razin (1999) and extends them to a closed economy. The use of the MPL condition shows why typical measures for welfare analysis that calculate the life-time taxes and expenditure are misleading.<sup>6</sup> Second, I show that the government can lead the economy to the (modified) golden rule level by accepting more immigrants within a finite time in a Pareto-improving way. This is in sharp contrast to the literature on the social security reform, in which it is argued that it is difficult to increase the capital stock through social security reform in a Pareto-improving way (Geanakoplos, Mitchell and Zeldes (1998)).

For the computational analysis, I add two important factors to the existing literature. The first is the possibility of Pareto improvement. When more immigrants are accepted, the wage rate falls and the interest rate increases initially. This generates winners and losers in the economy. In my calculation, I consider an immigration policy that Pareto-improves all generations. Taking Pareto improvement into consideration is important because if Pareto improvement is impossible to achieve, policy reform becomes less feasible even if it increases the over-all efficiency of an economy. The second factor is that I assume that the government surplus obtained by accepting more immigrants can be put into savings. When the government accepts more immigrants, the capital to labor ratio will decrease at first. However, due to the fact that the government can put its surplus into savings, the government can increase the capital to labor ratio in the long run. This will solve the problem of the under-accumulation of capital that results from operating a PYGO social security system.

### 3 The model

The model uses the standard overlapping generation model with a neoclassical production function developed by Diamond (1965). Each individual lives for two periods. When individuals are in the first period, they work and are called "young". When they are in the

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<sup>5</sup>Shimasawa and Oguro (2010) apply the model of Hehr, Joisch and Kotlikoff to the Japanese economy.

<sup>6</sup>To see why, see the discussion about Proposition 1

second period, they are retired and are called "old". I assume that immigrants come to the host country only when they are young and that the government of the host country prohibits the immigrants from immigrating when they are already old. Each individual of the native and the immigrant populations supplies one unit of labor inelastically in the host country when they are young.<sup>7</sup> I define individuals who are born at the beginning of period  $t$  in the host country as natives of cohort  $t$ , regardless of the nationality of the parents. Immigrants who move to the host country at the beginning of period  $t$  are considered as immigrants of cohort  $t$ . Let  $j$  be the index indicating nationality. If an individual is a native,  $j = n$  and if she or he is an immigrant,  $j = m$ . Let  $N_t^j$  be the number of the young of type  $j$  in period  $t$ . Let  $(c_t^{y,j}, c_{t+1}^{o,j})$  be the consumption in the young period and the old period of a type  $j$  ( $j = n, m$ ) individual of cohort  $t$ . Let  $g_t^{y,j}$  be the amount of publicly provided private goods, such as education and government provided health care service for the young, for each young individual of type  $j$  which is consumed at period  $t$ . Let  $g_t^{o,j}$  be the amount of publicly provided private goods, such as medicaid and publicly provided nursing home, for each old individual of type  $j$  at period  $t$ . Let  $g_t^{ind,j}$  be the amount of age-independent publicly provided private goods that are consumed by a young individual and an old individual of type  $j$ .<sup>8</sup> I assume that the utility function of cohort of type  $j$  is

$$U(c_t^{y,n}, g_t^y, g_t^{ind}, c_{t+1}^{o,n}, g_{t+1}^o, g_{t+1}^{ind}) = u^y(c_t^{y,j}, g_t^{y,j}, g_t^{ind,j}) + \frac{1}{1+\rho} [u^o(c_{t+1}^{o,j}, g_{t+1}^{o,j}, g_{t+1}^{ind,j})] . \quad (1)$$

I assume that  $u^i(c_t^{i,j}, g_t^{i,j}, g_t^{ind,j})$  ( $i = y, o$ ) are strictly increasing and concave functions. I assume additive separability of publicly provided private goods so that the provision of publicly provided private goods does not affect the consumption and saving decisions of individuals. This assumption simplifies the analysis because I also assume that the government redistributes the welfare gain of accepting more immigrants in the form of increased publicly provided private goods to individuals.<sup>9</sup>

For the production side, let  $F(L_t, K_t)$  be a production function where  $L_t$  and  $K_t$  are the total amount of labor and the total capital stock used at period  $t$ . Let  $\delta$  be the depreciation

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<sup>7</sup>In section 3.3, I show that the main results of this paper hold even when the labor supply is elastic. I use the assumption of an inelastic labor supply here to simplify the analysis in this section.

<sup>8</sup>In this paper, I ignore non-rivalry public goods. Note that the presence of non-rivalry public goods will favor immigration because accepting immigrants means that the cost of non-rivalry public goods will be shared by more individuals without decreasing their consumption.

<sup>9</sup>There are several ways to redistribute the welfare gain to individuals. I use this method to simplify the analysis. The main conclusion does not change when other way of redistributing the welfare gain are used.



rate of the capital. I assume that  $F(L_t, K_t)$  exhibits constant returns to scale and that both the marginal product of labor and capital are diminishing. I assume that the standard Inada condition is satisfied.

I assume that the economy is at the steady state initially and that the initial economy is dynamically efficient. In the literature, it is well-known that if the market interest rate is lower than the population growth rate, it is possible to Pareto-improve welfare (dynamic inefficiency). Since this paper's interest is not such a dynamic inefficiency problem, I postulate that at the initial steady state, the market interest rate is higher than the population growth rate (Cass(1972)). Furthermore, for the welfare analysis of accepting more immigrants, I make the following additional assumptions.

AS1: The amount of publicly provided private goods per person,  $(g_t^y, g_t^o, g_t^{ind})$  is constant at the initial steady state and  $(g_t^{y,j}, g_t^{o,j}, g_t^{ind,j}) = (g^{y,j}, g^{o,j}, g^{ind,j})$ .

AS2: The government uses a PYGO social security system at the initial steady state.

AS3: An immigrant has  $\phi \times 100$  percent efficient unit of labor in comparison with a native.

AS4: The government imposes the same flat wage tax rate on immigrants as on natives. This wage tax includes both social security tax and income tax. For the benefit of the social security, the government pays  $\phi \times 100$  percent of each native's benefit to each immigrant.

AS5: Immigrants and natives have the same preferences

AS6: The descendants of immigrants integrate with the native population and earn the same income as natives.

AS 7: Immigrants and their children stay permanently in the host country.

AS 8: The fertility rate of immigrants is equal to or higher than the fertility of the native.

AS 9: At the initial steady state, the government does not impose a distorting tax, such as capital income tax.

I use AS1 to focus on the issue of immigration, rather than on issues of public expenditure. AS2 is used to analyze the relationship between immigration and PYGO social security. AS 4 needs more discussion. Clearly, if the government can treat the immigrants in a discriminating way in the tax and the public pension systems, there is a way to increase the utility of both natives and immigrants. Normally, the wage rate of the immigrants in their country of origin is lower than the wage rate in their host country. Thus, it is possible to Pareto-improve the welfare of both natives and immigrants if (a) the government in the host country sets a high tax rate on incoming immigrants in such a way that the net wage rate of immigrants in their host country is still higher than their net wage rate of their

country of origin, (b) the government redistributes to natives the tax revenue collected from immigrants. AS4 precludes such an obvious case from occurring. AS5 is not important for the analysis. In section 3.3, AS5 is relaxed. AS 6-8 are not critical and are used to simplify the analysis. As for AS9, if there are already distorting taxes at the initial steady state in the economy, clearly the government can Pareto-improve the initial steady state by correcting those taxes. Because the labor supply is inelastic, the wage tax is optimal and the use of capital income tax is sub-optimal. I use AS9 because the focus of this paper is the first order effect of accepting immigrants, not tax reform. Of course, in reality, the governments of most developed countries impose capital income taxes. In the section 3.2, I show that the main conclusion does not change even in the presence of distorting taxes including capital income tax.

Let  $s_t^j, w_t^j$  and  $b_t^j$  be the saving, the pre-tax wage rate and the social security benefit of the old of type  $j$  of cohort  $t$ , respectively. Let  $r_t$ ,  $\tau_{wt}$  and  $\tau_{rt}$  be the pre-tax interest rate, the wage tax rate and the tax rate on interest income, respectively, at period  $t$ . Cohort  $t$  maximizes the above utility function subject to the budget constraint. The budget constraint of the cohort  $t$  of type  $j$  is

$$w_t^j(1 - \tau_{wt}) = c_t^{y,j} + s_t^j \text{ and } b_{t+1}^j + (1 + (1 - \tau_{rt})r_t)s_t^j = c_{t+1}^{o,j} . \quad (2)$$

Now consider the government budget constraint at period  $t$ . At this point, consider the government budget constraint with the capital income tax, even though in this section it is assumed that the capital income tax is not used at the initial steady state. The government budget constraint at period  $t$  is

$$\begin{aligned} \tau_{wt+1} \sum_{j=n,m} w_{t+1}^j N_{t+1}^j + \tau_{rt+1} r_{t+1} \sum_{j=n,m} s_t^j N_t^j - \sum_{j=n,m} N_t^j (b_t^j + g^{o,j} + g^{ind,j} - (1 + r_t)a_t) \\ - (g^{y,j} + g^{ind,j} - a_{t+1}) \sum_{j=n,m} N_{t+1}^j = 0 . \end{aligned} \quad (3)$$

where  $a_t$  is the government saving (debt) balance divided by the total number of population at  $t$ . Let  $F_K$  and  $F_L$  be the partial derivative of the production function with respect to capital and labor. Then,  $w_{t+1}^j$  and  $r_{t+1}$  are determined as follows:

$$\begin{aligned} w_{t+1}^n = F_L(N_{t+1}^n + \phi N_t^m, \sum s_t^j N_t^j + a_t \sum N_t^j), \quad w_{t+1}^m = \phi F_L(N_{t+1}^n + \phi N_t^m, \sum s_t^j N_t^j + a_t \sum N_t^j) \\ \text{and } r_{t+1} = F_K(N_{t+1}^n + \phi N_t^m, \sum s_t^j N_t^j + a_t \sum N_t^j) - \delta \end{aligned} \quad (4)$$

Substituting the individual budget constraints into the government budget constraint, we

have the following resource constraint:

$$\begin{aligned}
& F(N_{t+1}^n + \phi N_{t+1}^m, \sum s_t^j N_t^j + a_t \sum N_t^j) + (1 - \delta) \{ \sum s_t^j N_t^j + a_t \sum N_t^j \} \\
& = \sum \{ c_{t+1}^{y,j} + s_{t+1}^j + g^{y,j} + g^{ind,j} \} N_{t+1}^j + a_{t+1} \sum N_{t+1}^j + \sum N_t^j \{ c_{t+1}^{o,j} + g^{o,j} + g^{ind,j} \} \quad (5)
\end{aligned}$$

Let the fertility rate of natives and immigrants be  $\pi_n$  and  $\pi_m$ , respectively. Let  $\alpha_t$  be the immigrant-to-native ratio (INR). The immigration policy is expressed in terms of  $\alpha_t$ . For example, a one-time increase of the immigrant-to-native ratio means that  $\alpha_0 = \alpha^*$ ,  $\alpha_1 = \tilde{\alpha}$  and  $\alpha_t = \alpha^*$  for  $t \geq 2$  where  $\tilde{\alpha} > \alpha^*$ . Permanently increasing the immigrant-to-native ratio means that  $\alpha_0 = \alpha^*$  and  $\alpha_t = \tilde{\alpha}$  for all  $t \geq 1$  where  $\tilde{\alpha} > \alpha^*$ . The sum of natives and immigrants of cohort  $t$  is  $N_{t-1}^n \{ (1 + \pi_m) \times \alpha_{t-1} + (1 + \pi_n) \} \times (1 + \alpha_t)$ . At the steady state, the population growth rate is  $(1 + \pi_m) \times \alpha^* + (1 + \pi_n)$ . To avoid a situation in which the total population of period  $t - 1$  becomes zero or negative, I assume

$$(1 + \pi_m) \times \alpha_{t-1} + (1 + \pi_n) > 0 \quad \text{for } \alpha_{t-1} \geq 0. \quad (6)$$

### 3.1 The Effect of Increasing the INR through Intergenerational Channel

When more immigrants are accepted, they affect the welfare of natives in two channels. The first is the intergenerational channel. Immigrants will bring an inflow of labor that generates additional funding for PYGO social security. The second channel is the intra-redistribution effect. When the average earnings of immigrants are lower than the average earnings of natives but an immigrant consumes the same (or higher) amount of publicly provided private goods such as public education and health services as a native, the inflow of immigrants imposes a fiscal burden on the government budget and this tighter government budget is likely to reduce the welfare of natives. This section focuses on the inter-generational effect and its long run implication on capital accumulation. To see this intergenerational effect in a transparent way, I assume that the efficiency unit of labor of incoming immigrants is the same as that for natives and, that natives and immigrants consume publicly provided private goods in the same way. This implies that  $\phi = 1$  and  $g^{i,j} = g^i$  in our model. The case that includes both the inter-generational and the intra-redistribution effects is analyzed in section 3.5.

The assumptions, AS3-5,  $\phi = 1$  and  $g^{i,j} = g^i$  imply that consumption and labor supply of natives and immigrants are the same. It also implies that  $b^n = b^m$ . Thus, the resource

constraint becomes as follows:

$$F(\{(1 + \pi_m) \times \alpha_t + (1 + \pi_n)\} \times (1 + \alpha_{t+1}), (s_t^n + a_t)(1 + \alpha_t)) + (1 - \delta)(s_t^n + a_t) \geq \{c_{t+1}^{y,n} + s_{t+1}^n + g^y + g^{ind} + a_{t+1}\} \{(1 + \pi_m) \times \alpha_t + (1 + \pi_n)\} \times (1 + \alpha_{t+1}) + \{c_{t+1}^{o,n} + g^o + g^{ind}\}(1 + \alpha_t)$$

Before analyzing the effect of accepting more immigrants, we characterize the initial steady state. Let  $w^*$  and  $r^*$  be the pre-tax wage rate and the interest rate at the initial steady state respectively, where the immigration policy  $\alpha_t$  is equal to  $\alpha^*$  for all  $t$ . Let  $s^*$  and  $N_0^n$  be the savings of each individual and the number of old people at period 0. Let  $\tau_w^*$  and  $\tau_r^*$  be the wage tax rate and capital income tax rate at the initial steady state. Let  $b^*$  be the social security benefit at the initial steady state.

The initial steady-state economy with zero capital income tax and  $\alpha^*$  is characterized as follows:

$$s^* = \arg \max_{s_t^n} u^y(w^*(1 - \tau_w^*) - s_t^n) + v^y(g^y, g^{ind}) + \frac{1}{1 + \rho} [u^o((1 + r^*)s + b^*) + v^o(g^o, g^{ind})] \quad (7)$$

$$w^* = \frac{\partial F((1 + \pi_m)\alpha^* + 1 + \pi_n, s^*)}{\partial L}; \quad r^* + \delta = \frac{\partial F((1 + \pi_m)\alpha^* + 1 + \pi_n, s^*)}{\partial K}$$

$$(\tau_w^* w^* - g^y - g^{ind}) \times \{(1 + \pi_m)N_0^n \alpha^* + (1 + \pi_n)N_0^n\} \times (1 + \alpha^*) - (b^* + g^o) \times N_0^n (1 + \alpha^*) = 0$$

where  $\tau_w^*, g^y, g^o, g^{ind}, b^*$  are given.

The steady state level of the consumption and the utility with zero capital income tax are defined as follows:

$$c^{y*} \equiv w^*(1 - \tau_w^*) - s^*; \quad c^{o*} \equiv (1 + r^*)s^* + b^*; \quad u^* \equiv u^y(c^{y*}) + v^y(g^y, g^{ind}) + \frac{1}{1 + \rho} [u^o(c^{o*}) + v^o(g^o, g^{ind})]. \quad (8)$$

### 3.1.1 Welfare Effect of Increasing the INR

In this sub-subsection, I examine whether or not increasing the INR to a higher level permanently will Pareto-improve welfare. Increasing the INR permanently is defined as  $\alpha_0 = \alpha^*$  and  $\alpha_t = \tilde{\alpha}$  where  $\tilde{\alpha} > \alpha^*$  for  $t \geq 1$ . For the analysis, consider the following constrained maximization problem:

$$\begin{aligned}
V(\alpha) &= \max_{\{c_t^{y,n}, c_t^{o,n}, s_t^n, a_t | t=1,2,\dots\}} \frac{1}{1+\rho} [u^o(c_1^{o,n}) + v^o(g_o, g^{ind})] \\
\text{s.t. } & u^y(c_t^{y,n}) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho} [u^o(c_{t+1}^{o,n}) + v^o(g^o, g^{ind})] \geq u^* \quad \text{for } t = 1, 2, \dots \quad (9) \\
& F(\{(1+\pi_m)\alpha^* + (1+\pi_n)\} \times (1+\alpha), s^* \times (1+\alpha^*)) + (1-\delta)s^* \times (1+\alpha^*) \geq \\
& \{c_1^{y,n} + s_1^n + g^y + g^{ind} + a_1\} \{(1+\pi_m) \times \alpha^* + (1+\pi_n)\} \times (1+\alpha) + \{c_1^{o,n} + g^o + g^{ind}\} (1+\alpha^*) \quad (10) \\
& F((1+\pi_m)\alpha + (1+\pi_n), (s_t^n + a_t)) + (1-\delta)(s_t^n + a_t) \geq \\
& \{c_{t+1}^{y,n} + s_{t+1}^n + g^y + g^{ind} + a_{t+1}\} \times \{(1+\pi_m) \times \alpha + (1+\pi_n)\} + \{c_{t+1}^{o,n} + g^o + g^{ind}\} \quad \text{for } t = 1, 2, \dots \quad (11)
\end{aligned}$$

The above programming problem deserves several comments. First,  $V(\alpha)$  is the utility of cohort 0 at period 1 when the government accepts immigrants with a constant ratio  $\alpha^*$  at the initial steady state and starts to accept immigrants with ratio  $\alpha$  from period 1. Second, the first constraint is related to Pareto improvement and requires that all cohorts except cohort 0 need to have at least as the same utility as they would have at the initial steady state. Third, (10) and (11) are the resource constraints of period 1 and later periods.<sup>10</sup> I use the resource constraint, rather than the government budget constraint, because the two are equivalent.<sup>11</sup>

Let  $\gamma_t$  be the Lagrangian multiplier of (9). Let  $L$  and  $\lambda_t$  be the Lagrangian function and Lagrangian multiplier of the resource constraints of (10) and (11), respectively. Let  $\gamma_t^*$  and  $\lambda_t^*$  be the Lagrangian multiplier of the utility constraint and the resource constraint at period  $t$  when  $\alpha = \alpha^*$ . Then, we have the following observation.

**Observation 1**

*When  $\alpha = \alpha^*$  the solution of MPP is*

$$c_t^{y,n} = c^{y*}, \quad c_0^{o,n} = c^{o*}, \quad s_t = s^*, \quad a_t = 0 \quad \text{for } t = 1, 2, \dots \quad (12)$$

$$\lambda_1^* = \frac{1}{1+\rho} u'_o(c_1^o) \quad \text{and} \quad \lambda_{t+1}^* = \frac{(1+\pi_m)\alpha^* + 1 + \pi_n}{1+r^*} \lambda_t^* \quad (13)$$

$$\gamma_t^* = \frac{1}{u'_y(c^{y*})} \lambda_t^* \{(1+\pi_m)\alpha^* + 1 + \pi_n\} \quad \text{and for } t = 1, 2, \dots \quad (14)$$

For the proof of observation 1, see appendix B1.

<sup>10</sup>In (11),  $(1+\alpha)$  is multiplied on both sides of the original resource constraint. Thus, it is eliminated.

<sup>11</sup>In these resource constraints, tax and social security are not defined. However, once the consumption and saving level are defined, the tax and social security benefit are calculated implicitly.

Observation 1 implies that when  $\alpha = \alpha^*$ , the initial steady state allocation is Pareto-efficient and it is not possible to have Pareto improvement from the initial steady state. Now suppose that the government increases the INR from  $\alpha^*$ . Whether or not such an increase of the INR Pareto-improves welfare can be analyzed by calculating  $V'(\alpha)$  and evaluating it at  $\alpha = \alpha^*$ . From the envelope theorem,  $dV/d\alpha|_{\alpha=\alpha^*}$  is equal to

$$\left\{ \{(1 + \pi_m)\alpha^* + 1 + \pi_n\} \lambda_1^* + (1 + \pi_m) \sum_{t=2}^{\infty} \lambda_t^* \right\} \times \left\{ F_L((1 + \pi_m)\alpha^* + 1 + \pi_n, s^*) - c^{y^*} - s^* - g^y - g^{ind} \right\}. \quad (15)$$

The first bracket is positive because the Lagrangian multiplier of the resource constraint is positive. In the second bracket, the first term is the marginal product of labor, which is what an individual contributes to the economy when he or she is young at the initial steady state.  $c^{y^*} + s^*$  is equal to the after tax income of the young individual.  $g^y + g^{ind}$  are the private goods that are provided by the government for the young individual. Therefore,  $c^{y^*} + s^* + g^y + g^{ind}$  is what an individual receives while he or she is young at the initial steady state. Therefore, the bracket yields the amount of an intergenerational transfer from the young to the old. For example, if the inside of the second bracket is equal to zero, it implies that what the young earn is allocated fully to the young (by including publicly provided private goods), whereas values above zero entail that there is an intergenerational transfer.

Because the marginal product of labor of the young is the pre-tax wage of the young at the initial steady state and because  $c^{y^*} + s^*$  is the after-tax income of the young by the definition,  $F_L(1 + \pi_n, s^*) - c^{y^*} - s^*$  is the amount of tax paid by the young. This implies that if the amount of tax paid by the young is greater than the goods provided by the government to the young at the initial steady state, accepting more immigrants Pareto-improves welfare.<sup>12</sup>

**Proposition 1. (MPL condition)** *If there exist intergenerational transfers from the young to the old at the initial steady state in the sense that the marginal product of labor of the young is greater than the amount of resources which they receives while they are young (after tax income plus publicly provided private goods), then accepting more immigrants Pareto-improves the welfare of all generations. Alternatively, if the amount of tax that the young pay is greater than the amount of publicly provided private goods for the young,*

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<sup>12</sup>Note that this condition does not change even in the presence of public goods because an increase of the number of immigrants does not affect the consumption of public goods by the nature of public goods.

*accepting more immigrants Pareto-improves the welfare of all generations.*

It should be noted that this MPL condition is important for measuring the real effect on welfare of accepting more immigrants. In a typical study of assessing the effect on welfare of accepting more immigrants, it calculates the present value of the government expenditure and the tax revenue that a typical immigrant brings. The above MPL condition shows that such a welfare index is quite misleading. The above MPL condition suggests that the difference of taxes and expenditure while immigrants are young is more important than the present value of taxes and expenditures over the life cycle.<sup>13</sup>

Graphically, Proposition 1 can be explained as follows. Consider a simple case in which the depreciation rate is 100% ( $\delta = 1$ ) and there are no immigrants at the initial steady state ( $\alpha^* = 0$ ). Assume that the population of the old is equal to one. At period 0, the following resource constraint must hold:

$$F(1 + \pi_n, s^*) - (c^{o*} + g^o + g^{ind}) - (1 + \pi_n)(c^{y*} + s^* + g^y + g^{ind}) = 0. \quad (16)$$

Note that  $F(1 + \pi_n, s^*)$  is the GDP per one old individual when the population growth rate is equal to  $\pi_n$ . Now consider the graph of  $(y, x)$  where the vertical axis measures the GDP per one old individual and the horizontal axis measures one plus the population growth rate. Thus,  $x = 1 + \pi_n$ ,  $y = F(x, s^*)$ . Next, draw the line of  $(y, x)$  where  $y$  is defined as  $y = (c^{y*} + s^* + g^y + g^{ind})x$ . At  $x = 1 + \pi_n$ , the vertical distance of this line represents the total amount of resources used for the young divided by the number of old individuals when the population growth rate is equal to  $\pi_n$ . Thus, the difference between  $y = F(x, s^*)$  and  $y = (c^{y*} + s^* + g^y + g^{ind})x$  at  $x = 1 + \pi_n$  represents the amount of resources used for one old individual.

Now suppose that a social planner increases the population growth rate by accepting immigrants permanently. This implies that  $x$  moves to the right from  $x = 1 + \pi_n$ . If the slope of  $y = F(x, s^*)$  at the  $x = 1 + \pi_n$  is greater than  $c^{y*} + s^* + g^y + g^{ind}$ , the social planner can maintain the same allocation of resources per each young individual (private consumption, savings, government provided private goods and age-independent public goods) and increase the allocation of resources to each old individual. Clearly this constitutes Pareto improvement. Note that when the government accepts immigrants there is a surplus that is equal to  $\partial F(1 + \pi_n, s^*)/\partial L - (c^{y*} + s^* + g^y + g^{ind})$  in every period. Also

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<sup>13</sup>When immigrants and natives are different, then the sum of the redistribution from immigrants to natives and the difference of MPL and what an individual receives while she is young becomes important. See section 3.3.

note that  $\sum \lambda_t^*$  is the present discounted value of the increased amount of resources for all periods. Therefore, (15) presents the present discounted value of the surplus of all periods that is obtained by increasing the population growth rate(see Figure A1).

### 3.2 Elastic labor Supply, Distorting Taxes and Implementation of Pareto improvement

In the preceding subsection, it was shown that it is Pareto-improving to accept more immigrants when there are intergenerational transfers from the young to the old in the sense that the marginal product of labor of the young is greater than the resources that are allocated to the young. With a neoclassical production function that exhibits diminishing marginal product, the pre-tax wage decreases and the pre-tax interest rate increases when more immigrants are accepted. To Pareto-improve the welfare of all generations, taxes must be adjusted or other policies must be implemented. However, the preceding analysis does not show how taxes or other policies are used while more immigrants are being accepted.

In addition, in the preceding subsection, I assumed that the labor supply is inelastic and that there is no capital income tax at the initial steady state. When the labor supply is elastic, the wage tax rate or zero capital income tax rate can be suboptimal at the initial steady state. When the wage tax or capital income tax is sub-optimal. The analysis of the preceding subsection can not be used because the earlier analysis assumed that the initial steady state is efficient.<sup>14</sup>

In this subsection, I will show that, as long as the MPL condition is satisfied, increasing the INR Pareto-improves welfare without relying on changing incentives of individuals even if the labor supply is elastic and even if there are distorting taxes (wage tax rate or capital income rate or both) at the initial steady state.<sup>15</sup> In addition, I show that a relatively simple adjustment of taxes and social security benefit achieves this Pareto improvement while the government is increasing the INR.

For the analysis, let  $\tilde{\alpha}$  be a time invariant new immigration policy from period 1 where  $\tilde{\alpha} > \alpha^*$ . I assume that the labor supply is elastic and that the government uses a capital income tax and a wage tax at the initial steady state. I assume that those taxes do not

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<sup>14</sup>If we assume that the wage tax rate and capital income tax rate are the second-best, we can apply the analysis of the preceding subsection to the case in which labor supply is endogenous. However such an assumption is very restrictive.

<sup>15</sup>In the presence of non-optimal taxation, it is obvious that it is possible to Pareto-improve welfare if the government can induce changes in incentives of individuals.



need to be the second best optimal. With the assumption of the elastic labor supply, the individual budget constraint (2) is modified as follows:<sup>16</sup>

$$w_t^j l_t^j (1 - \tau_{wt}) = c_t^{y,j} + s_t^j \text{ and } b_{t+1}^j + (1 + (1 - \tau_{rt})r_t)s_t^j = c_{t+1}^{o,j}$$

Let  $l^*$  be the labor supply of a young native or immigrant when he or she faces the same wage rate and tax rate as at the initial steady state. I assume that at the initial steady state the social security benefit is proportional to pre-tax earnings:

$$b^* = \Omega \times w^* l^* \tag{17}$$

When the government increases the INR, the wage rate falls and the interest rate increases initially. To achieve Pareto improvement by increasing the INR, first I assume that the government sets the tax rates such that after-tax wage and interest rate after an increase of the INR are equal to the after-tax wage rate and interest rate at the initial steady state. Then, the wage tax rate and the interest tax rate for period  $t$  are set as follows:

$$w_t(1 - \tau_{wt}) = w^*(1 - \tau_w^*) \text{ and } r_t(1 - \tau_{rt}) = r^*(1 - \tau_r^*) \tag{18}$$

Second, I assume that the government re-scales  $\Omega$  so that the social security benefit becomes proportional to after-tax earnings, not pre-tax earnings, and that an individual receives the same benefit when wage rate, wage tax rate and labor supply are at the same level as at the initial steady state. This implies that

$$b_{t+1}^j = \frac{\Omega}{1 - \tau_w^*} w_t(1 - \tau_{wt}) l_t^j \tag{19}$$

Note that  $b_{t+1}^j = \Omega w^* l_t^j$  when  $w_t(1 - \tau_{wt}) = w^*(1 - \tau_w^*)$ .

When the government sets taxes and social security benefit in this way, saving behavior and labor supply behavior do not change because the budget constraint of a consumer at any period  $t$  is the same as at the initial steady state. If the government provides at least as the same level of publicly provided private goods, the levels of the utility of all cohorts are at least as the same as at the initial steady state.

As for the extent of the increased immigrants, motivated by Proposition 1, I assume that the MPL condition is satisfied at the initial steady state:

$$l^* \frac{\partial F(l^*((1 + \pi_m)\alpha^* + 1 + \pi_n), s^*)}{\partial L} > c^{y*} + s^* + g^y + g^{ind} \tag{20}$$

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<sup>16</sup>In the following budget constraint, I put superscript  $j$  to apply the current analysis to other cases although it is assumed that natives and immigrants behave in the same way since they have the same preferences and wage levels.

The LHS of the above equation is a marginal increase of the output due to a one unit increase of the population. The RHS is the amount of resources that an individual receives when he or she is young.

The result that was generated in the preceding subsection shows that a marginal increase in the number of immigrants Pareto-improves welfare if the MPL condition is satisfied. However, that result does not imply that an unlimited acceptance of immigrants always Pareto-improves welfare. I assume that when the new immigration policy  $\tilde{\alpha}$  is operative, the marginal increase of the output due to a one unit increase of the population is greater or equal to the amount of resources that an individual receives when he or she is young at the initial steady state. This implies that

$$l^* \frac{\partial F(l^* \times (1 + \tilde{\alpha}) \times \{(1 + \pi_n)N_{t-1}^n + (1 + \pi_m) \times N_{t-1}^m\}, (N_{t-1}^n + N_{t-1}^m)s^*)}{\partial L} \geq c^{y^*} + s^* + g^y + g^{ind} . \quad (21)$$

Note that  $(1 + \pi_m) \times N_{t-1}^m + (1 + \pi_n)N_{t-1}^n$  is the population of young native at period  $t$ . Using the homogeneity of the production function and  $N_{t-1}^m = \tilde{\alpha}N_{t-1}^n$ , (21) can be written as

$$l^* \frac{\partial F(l^* \times \{(1 + \pi_n) + (1 + \pi_m) \times \tilde{\alpha}\}, s^*)}{\partial L} \geq c^{y^*} + s^* + g^y + g^{ind} . \quad (22)$$

When the government accepts more immigrants according to (20) and (21), and it adjusts the wage tax rate and capital income tax rate so that the after-tax wage rate and after-tax interest rate are the same as at the initial steady state as defined in (18), the government can accumulate savings as I show below. This implies that it is possible that, at some point, the marginal product of capital (MPK) becomes equal to the golden level of capital stock per capita. But when the MPK becomes equal to the golden rule level, it is clearly better to use the entire government surplus to increase the supply of publicly provided private goods rather than to increase the government savings balance. Thus, I assume that as long as the MPK is higher than the golden rule level, the government uses some of the government budget surplus to increase the government saving balance and the remainder to increase the supply of publicly provided private goods. When the MPK reaches the golden rule level, the government uses all of the surplus to increase the publicly provided private goods. Thus, we have the following MPK condition:

$$\frac{\partial F}{\partial K} \geq \delta + (1 + \pi_m)\tilde{\alpha} + \pi_n \quad (23)$$

Now we examine whether social security benefit and taxes determined by (19) and (18)

are feasible from the point of the government budget constraint. To check the feasibility of such taxes, consider the net government budget surplus for period 1,  $SP_1$ .

$$SP_1 = w_1 \tau_{w1} l^* \sum_{j=n,m} N_1^j + r_1 \tau_{r1} s^* \sum_{j=n,m} N_0^j - (b^* + g^o + g^{ind}) \sum_{j=n,m} N_0^j - (g^y + g^{ind}) \sum_{j=n,m} N_1^j \quad (24)$$

Note that  $\tau_{w1}$  and  $\tau_{r1}$  are defined in (18). By substituting  $\tau_{w1}$  and  $\tau_{r1}$  into  $SP_1$  and using the homogeneity of the production function, we have (see appendix B2)

$$SP_1 = \frac{1}{l^*} \int_{l^* N_1^n(1+\alpha^*)}^{l^* N_1^n(1+\tilde{\alpha})} [l^* \frac{\partial F(z, s N_0^n(1+\alpha^*))}{\partial L} - c^{y*} + s^* - (g^y + g^{ind})] dz \quad (25)$$

where  $N_1^n = N_0^n(1+\pi_n) + N_0^n(1+\alpha^*)(1+\pi_m)$ . Thus, from the MPL conditions (20) and (21), the inside of the integration is positive for  $z \in [l^* N_1^n(1+\alpha^*), l^* N_1^n(1+\tilde{\alpha})]$ . This means that this tax plan is feasible in period 1. The government can use some of the surplus of the budget to increase the supply of publicly provided private goods and put remainder in savings. Let  $a_1 > 0$  be the balance of the government savings per each young individual at the end of period 1. What will be the net budget surplus in period 2,  $SP_2$  ?

Note that the natives and immigrants of cohort 1 will save the same as the natives and immigrants at the initial steady state, because they face the same after-tax wage rate and interest rate under the proposed tax policy as at the initial steady state. This implies that  $s_1^n = s^*$  for both groups of cohort 1. The total capital stock for period 2 is  $\sum_j N_1^j (s^* + a_1)$ . Thus,  $SP_2$  becomes

$$SP_2 = \sum_{j=n,m} N_2^j \times (w_2 \tau_{w2} - g^y - g^{ind}) + \sum_{j=n,m} N_1^j \times (r_2 \tau_{r2} s^* - b^* - g^o - g^{ind}) + (1+r_2)a_1 \sum_{j=n,m} N_1^j. \quad (26)$$

Note that the pre-tax wage for period 2,  $w_2$ , and the pre-tax interest rate for period 2,  $r_2$ , are equal to

$$w_2 \equiv \frac{\partial F(l^*(N_2^n + N_2^m), (N_1^n + N_1^m)(s^* + a_1))}{\partial L} \quad \text{and} \quad r_2 \equiv \frac{\partial F(l^*(N_2^n + N_2^m), (N_1^n + N_1^m)(s^* + a_1))}{\partial K} - \delta \quad (27)$$

Again the government sets  $\tau_{w2}$  and  $\tau_{r2}$  such that after-tax wage rate and after-tax interest rate become the same as at the initial steady state. Thus,  $SP_2$  becomes (see appendix B3)

$$SP_2 = \int_{s^* N_1^n(1+\tilde{\alpha})}^{(s^*+a_1)N_1^n(1+\tilde{\alpha})} \left\{ \frac{\partial F(l^* N_2^n(1+\tilde{\alpha}), z)}{\partial K} + (1-\delta) \right\} dz \\ + (1+\tilde{\alpha}) N_1^n (1+\pi_m) \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F(l^* \{1+\pi_n + (1+\pi_m)z\}, s^*)}{\partial L} l^* - (1-\tau_w^*) w^* - (g^y + g^{ind}) \right] dz \quad (28)$$

The first term of (28) measures the welfare gain that arises from the additional savings that the government accumulated at the end of period 1. The second term measures the welfare gain that arises from the increased population growth rate in the presence of the PYGO social security. From the MPK condition (23), the inside of the first integration is positive. From (22), the inside of the second integration is positive. Thus,  $SP_2$  is positive and the government can implement the proposed tax policy. Again, at the end of period 2, the government can use some of the above surplus to increase the supply of publicly provided private goods and put the rest to increase the balance of government savings. Similarly, the government surplus for period  $t$  becomes

$$\begin{aligned}
SP_t = & \int_{s^* N_{t-1}^n (1+\tilde{\alpha})}^{(s^*+a_{t-1}) N_{t-1}^n (1+\tilde{\alpha})} \left\{ \frac{\partial F(l^* N_t^n (1+\tilde{\alpha}), z)}{\partial K} + (1-\delta) \right\} dz \\
& + (1+\tilde{\alpha}) N_t^n (1+\pi_m) \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F((l^* \{1+\pi_n + (1+\pi_m)z\}, s^*))}{\partial L} l^* - (1-\tau_w^*) w^* - (g^y + g^{ind}) \right] dz
\end{aligned} \tag{29}$$

where  $a_t$  is the balance of government saving per each young individual at the end of period  $t-1$ . Again the government uses some of the surplus for increasing publicly supplied private goods and puts the rest in savings. This implies that  $SP_t > 0$  for all  $t = 1, 2, \dots$ . Thus, we have the following proposition:

**Proposition 2.** *Consider an economy in which the labor supply is elastic and the (optimal or non-optimal) wage and interest taxes are used at the initial steady state. If the MPL condition is satisfied at the initial steady state, accepting more immigrants with tax rule (18) and social security benefit rule (19) Pareto-improves the welfare of all generations.*

### 3.2.1 Government Savings and the Golden Rule

In this subsection, I examine the capital stock path and government saving path when the government increases the INR. To examine the government saving path, I need to specify how much of the government surplus,  $SP_t$ , is put into savings. To simplify the argument, I assume that the surplus that arises from the increased government savings at period  $t-1$ , which is the first integration of  $SP_t$ , is put into the government savings at period  $t$ . Note that the government could use some part of the second integration in  $SP_t$ , the surplus that is generated directly from the increased immigration. Thus, my assumption is a conservative value for the government savings. I will show that, even with this conservative level, the economy reaches the golden rule level of capital stock per capita within a finite time in a

Pareto-improving way. The government savings per each young individual at the end of period  $t$  for  $t \geq 2$ ,  $a_t$ , becomes

$$a_t = \frac{1}{N_t^n(1 + \tilde{\alpha})} \int_{s^* N_{t-1}^n(1 + \tilde{\alpha})}^{(s^* + a_{t-1})N_{t-1}^n(1 + \tilde{\alpha})} \left\{ \frac{\partial F(l^* N_t^n(1 + \tilde{\alpha}), z)}{\partial K} + (1 - \delta) \right\} dz \quad (30)$$

Note that from (23), we have

$$a_t \geq \frac{1}{N_t^n(1 + \tilde{\alpha})} \int_{s^* N_{t-1}^n(1 + \tilde{\alpha})}^{(s^* + a_{t-1})N_{t-1}^n(1 + \tilde{\alpha})} [(1 + \pi_m)\tilde{\alpha} + 1 + \pi_n] dz = a_{t-1}$$

Thus,  $a_t$  is increasing over time as long as  $a_t$  is determined according to equation (30) and (23). By rewriting the above equation, we have

$$a_t = \frac{1}{1 + \pi_n + \tilde{\alpha}(1 + \pi_m)} \{ F(l^* \{1 + \pi_n + \tilde{\alpha}(1 + \pi_m)\}, s^* + a_{t-1}) - F(l^* \{1 + \pi_n + \tilde{\alpha}(1 + \pi_m)\}, s^*) + (1 - \delta)a_{t-1} \} \equiv Q(a_{t-1})$$

Now, consider the graph of  $a_t = Q(a_{t-1})$ .  $Q(a_{t-1})$  is zero at  $a_{t-1} = 0$  and  $Q(a_{t-1})$  is concave due to the diminishing marginal product of capital. The slope of  $Q(a_{t-1})$  at  $a_{t-1} = 0$  is

$$Q'(0) = \frac{1}{1 + \pi_n + \tilde{\alpha}(1 + \pi_m)} \left( \frac{\partial F(l^* \{1 + \pi_n + \tilde{\alpha}(1 + \pi_m)\}, s^*)}{\partial K} + 1 - \delta \right) \quad (31)$$

Because of assumption (23),  $Q'(a_{t-1})$  at  $a_{t-1} = 0$  is greater than 1. Thus, the  $a_t = Q(a_{t-1})$  and 45 degree line intersect at  $a_{t-1} = 0$  and at  $a^*$  where  $a^* > 0$ . Let  $a^{**}$  be the point where  $Q'(a^{**}) = 1$ . This implies that at  $a^{**}$

$$\frac{\partial F(l^* \{1 + \pi_n + \tilde{\alpha}(1 + \pi_m)\}, s^* + a^{**})}{\partial K} + 1 - \delta = 1 + \pi_n + \tilde{\alpha}(1 + \pi_m).$$

In other words, at  $a^{**}$  the golden rule is satisfied. Note that the government can choose  $a_1$  because the surplus at period 1 is strictly positive. From the graph of  $a_t = Q(a_{t-1})$ ,  $a_t$  keeps increasing starting from a small  $a_1 > 0$ . Before it reaches  $a^*$ , it reaches  $a^{**}$  within a finite time. This implies that the economy reaches the golden rule level of capital stock per capita within a finite time.

Note first that in this analysis, I assume that only the first integration of  $SP_t$ , i.e. the surplus that arises from the increased government savings at period  $t - 1$ , is put into savings at period  $t$ . However, the second integration of  $SP_t$ , i.e. the surplus which arises directly from increased immigrants, also can be put into savings. Thus, the government can shorten the time to reach the golden rule level by putting the surplus that arises directly from increased immigrants. Secondly, we can note that the government can induce the

economy to reach the modified golden rule level within a finite time in a Pareto-improving way, because the modified golden rule level is lower than the golden rule level.

**Proposition 3. (Reaching (modified) Golden Rule Level)** *Suppose that a PYGO social security system is used initially and that the MPL condition is satisfied. Then, by accepting more immigrants and using the proposed tax rule, social security benefit rule and government saving policy, the government can induce the economy to reach the (modified) golden rule level within in a finite time in a Pareto-improving way.*

### 3.3 Intra-redistributional Channel and Difference of Productivities and Preferences

When immigrants earn less than natives or immigrants consume more publicly provided private goods than natives, accepting more immigrants could decrease the welfare of natives, because it means that more resources are taken from natives and used for immigrants. This is the intra-redistributional channel of accepting more immigrants. A similar redistribution could also happen when the preferences of immigrants and natives are different and the labor supply or savings of immigrants differs from those of natives. To analyze this redistributional channel of accepting more immigrants, consider again a permanent change of immigration policy such that  $\tilde{\alpha} > \alpha^*$ . Let  $(c^{y,j*}, c^{o,j*}, s^{j*}, l^{j*})$ , be the consumption at the young period, the consumption at the old period, savings and labor supply of type  $j$  ( $j = n, m$ ), respectively when the wage rate and the interest rate are at the initial steady-state level and the tax rate on interest and wage are also at the initial steady state level. Since I assume that the preferences and the productivities of the native and immigrant are different in this section, I put superscript  $j$  even for the variables at the steady-state situation. Also, let  $g^{i,j}$  be the publicly provided private goods for goods type  $i$  and type  $j$  nationality where  $i = y, o, ind$  and  $j = n, m$ . For conditions that guarantee that accepting more immigrants

Pareto-improves welfare, I assume that the following condition is satisfied:

$$\begin{aligned}
\text{Modified MPL condition} &= \left[ \frac{\partial F}{\partial L} l^{n*} - c^{y,n*} - s^{n*} - g^{y,n} - g^{ind,n} \right] \\
&\quad + \alpha \left[ \frac{\partial F}{\partial L} \phi l^{m*} - c^{y,m*} - s^{m*} - g^{y,m} - g^{ind,m} \right] \\
&\quad (1 + \pi_n + (1 + \pi_m)\alpha) \times \frac{\partial F}{\partial L} \phi l^m + \left[ \frac{\partial F}{\partial K} + (1 - \delta) \right] s^{m*} \\
&\quad - (1 + \pi_n + (1 + \pi_m)\alpha) [\phi(1 - \tau_w^*) w^* l^{m*} + g^{y,m} + g^{ind,m}] \\
&\quad - (c^{o,m*} + g^{o,m} + g^{ind,m}) > 0 \text{ for all } \alpha \in [\alpha^*, \tilde{\alpha}] : \\
\text{where } \frac{\partial F}{\partial L} &= \frac{\partial F((1 + \pi_n + \alpha(1 + \pi_m))l^n + \phi J(\alpha)l^m, s^{n*}N_t^n + s^{m*}\alpha)}{\partial L}, \\
\frac{\partial F}{\partial K} &= \frac{\partial F((1 + \pi_n + \alpha(1 + \pi_m))l^n + \phi J(\alpha)l^m, s^{n*}N_t^n + s^{m*}\alpha)}{\partial K} \\
&\quad \text{and } J(\alpha) = (1 + \pi_n + \alpha(1 + \pi_m))\alpha \tag{32}
\end{aligned}$$

The first line is the marginal product of labor of a native minus the amount of resources that a young native receives while he/she is young. The inside of the bracket of the second line is the marginal product of labor of an immigrant minus the amount of resources that an immigrant receives while he/she is young. Thus, the first and the second lines are the intergenerational effect. The third, fourth and fifth lines comprise what young and old immigrants earn minus what young and old immigrants receive. Thus, the third to fifth lines show the redistribution effect. If a native and an immigrant are completely identical with respect to human capital level, labor supply, savings and the consumption of publicly provided private goods, the sum of the third to fifth line is equal to zero. (32) shows that the sum of intergenerational effect and redistributive effect should be greater than zero. When immigrants and natives are identical with respect to human capital level, labor supply, savings and consumption of publicly provided private goods, (32) becomes (21). Given (32), we have the following propositions.

**Proposition 4** *Assume that native and immigrant are not identical with respect to human capital level, labor supply, savings and consumption of publicly provided private goods. If the Modified MPL condition is satisfied, it is possible to Pareto-improve the welfare of all generations by accepting more immigrants. For the proof, see Appendix B4.*

**Proposition 5** *If the Modified MPL condition is satisfied, the government can induce the economy to reach the (modified) golden rule level within a finite time in a Pareto-improving way by accepting more immigrants. For the proof, see Appendix B5.*

## 4 Quantifying the Welfare Gain of Accepting More Immigrants in the Presence of a PYGO Social Security System

Propositions 1 and Proposition 2 above show that accepting more immigrants can Pareto-improve the welfare of all generations if there are intergenerational transfers in the sense that the marginal product of labor of a young individual is greater than what he or she receives including publicly provided private goods while he or she is young (the MPL condition). Furthermore, Proposition 3 shows that if the government can put some of the welfare gain into savings, it is possible to induce the economy to reach the golden rule level of capital in a Pareto-improving way within a finite time. This result is in sharp contrast to findings in the literature on social security reform which shows that some generation must bear a double burden to increase the capital stock of the economy in the presence of a PYGO social security system (Geanakoplos, Mitchell and Zeldes 1998). In addition, the proposition 4 and 5 show that even if an immigrant earns less than a native, accepting more immigrants Pareto-improves welfare when the intra-redistribution is not as large as the intergenerational transfer from the young to the old.

A number of issues arise as a result of accepting proposition 1-5. First, those propositions are based on a two-period overlapping generation model. In a realistic multi-period overlapping generation model, it might not be possible for an economy to reach the (modified) golden rule level of capital stock per capita within a finite time in a Pareto-improving way by accepting more immigrants. Second, although proposition 3 shows that the economy reaches the (modified) golden rule level of capital stock per capita within a finite time, in practice it might take a long time, as long as 1000 years, to reach the golden rule level. Third, propositions 1-5 are silent on the quantitative effect on welfare. Given that it might take a quite long time to reach the golden rule level, the welfare gain of accepting more immigrants can be very small. In addition, propositions 4 and 5 say nothing about the degree to which the difference in productivity between immigrants and natives is allowed for with respect to Pareto improvement when more immigrants are accepted.

This section addresses these issues. To address these issues, I use the computational overlapping generation model developed by Auerbach and Kotlikoff (1987). I assume that the model economy consists of overlapping generations in which each generation lives for 80 periods and that the probability of death increases with each passing period. I assume that the model economy is similar to the US economy in several dimensions.<sup>17</sup> In the

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<sup>17</sup>However, the model economy is different in several important dimensions as well. For example, the



analysis, I assume that initially the model economy is on the balanced growth path and the percentage of immigrants to natives (PITN), which is defined as 100 times the number of immigrants divided by the number of natives, is similar to that which the US 2000 census data indicates. Then, I will examine whether it is possible to Pareto-improve the welfare of all the generations in the model economy by increasing the PITN by a reasonable size. In addition, I examine how long it takes for the model economy to reach the (modified) golden-rule level in a Pareto-improving way and quantify the Pareto-improving welfare gain. To check the robustness, I recalculate the model by changing the value of the following parameters: the replacement, initial government debt (asset) level, the level of earnings of immigrants, immigrants' consumption of publicly provided private goods, probability of immigrants returning to the country of origin, the CRRA and the time preference rate.

#### 4.1 The Model Economy: Auerbach and Kotlikoff Model with Immigration

Agents will show up in the model from age 1. I assume that age 1 corresponds to age 20 in real life. They work from age 1 until age 45. From the beginning of age 46, they retire. At each age, they die with some probability and they can live until age 80. Let  $i$  be the index of age. For  $i \geq 2$ , let  $p_i$  be the probability that an agent is alive at the age  $i$ , given that he or she is alive until age  $i - 1$ . Due to the lack of data, I assume that  $p_i$  is the same for natives and immigrants. To simplify the notation, I assume that  $p_1 = 1$ .<sup>18</sup> An agent who enters the model at period  $t$  maximizes the following utility function:

$$\max \sum_{i=1}^{45} \beta^i \prod_{q=1}^i p_q \left\{ \frac{[(c_{t-1+i}^{i,j})^\zeta (1 - l_{t-1+i}^{i,j})^{1-\zeta}]^{1-\gamma}}{1-\gamma} + g_{t-1+i}^{ij} \right\} + \sum_{i=46}^{80} \beta^i \prod_{q=1}^i p_q \left\{ \frac{[c_{t-1+i}^{i,j}]^{\zeta(1-\gamma)}}{1-\gamma} + g_{t+1-i}^{ij} \right\} \quad (33)$$

where  $c_{t-1+i}^{i,j}$  and  $l_{t+1-i}^{i,j}$  are the amounts of private consumption and labor supply of type  $j$  agent of age  $i$  at period  $t - 1 + i$ .  $g_{t+1-i}^{ij}$  is the amount of publicly provided private goods to age  $i$  agent in period  $t + 1 - i$ . I assume that the amount of  $g_{t+1-i}^{ij}$  is chosen by the government. In this formulation, as in Storesletten (2000), I postulate that the immigrants

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model economy does not include a certain aspect of open economy such as international trade and capital mobility. The model economy does not incorporate the accumulation of human capital of the natives and immigrants.

<sup>18</sup>Infant and child mortality is defined in (39).

assume they will stay in the host country until the end of their lives.<sup>19</sup> Let  $s_t^{i,j}$  be the savings of type  $j$  of age  $i$  at time  $t$ . The budget constraint of an agent at age  $i$  is

$$s_{t-2+i}^{i-1,j}(1+r_{t-1+i}(1-\tau_{r,t-1+i})) + (1-\tau_{w,t-1+i})w_{t-1+i}H^{i,j} \times l_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 1 \leq i \leq 45 \quad (34)$$

$$s_{t-2+i}^{i-1,j}(1+r_{t-1+i}(1-\tau_{r,t-1+i})) + b_{t-1+i}^{i,j} = c_{t-1+i}^{i,j} + s_{t-1+i}^{i,j} \text{ for } 46 \leq i \leq 80 \quad (35)$$

$$s_{t-1+i}^{i,j} = 0 \text{ for } i = 0 \text{ and } s_{t-1+i}^{i,j} \geq 0 \text{ for } 1 \leq i \leq 80 \quad (36)$$

where  $H^{i,j}$  is the efficient unit of human capital of type  $j$  at age  $i$  and  $H^{i,j} > 0$  for  $1 \leq i \leq 45$  and  $H^{i,j} = 0$  for  $i \geq 46$ .  $w_t$  is the wage rate for one efficient unit of labor at period  $t$ . I assume that an individual cannot have a negative savings balance. Once an individual dies, the government imposes a 100 percent inheritance tax.  $b_{t-1+i}^{i,j}$  is the social security benefit for type  $j$  of age  $i$  that is given at period  $t-1+i$ . For  $i \leq 45$ ,  $b_{t-1+i}^{i,j} = 0$  and for  $i \geq 46$ ,  $b_{t-1+i}^{i,j}$  is determined as follows:

$$b_{t-1+i}^{i,j} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45} (1+\mu)^{45-i} w_{t-1+i} l_{t-1+i}^{i,j} H^{i,j}}{45 \times 12}$$

where  $RR$  is the replacement and  $AIME^j(t)$  is the average income monthly index of the cohort who become age 1 at time  $t$ .

For the production function, I assume that the economy's aggregate production can be described by the Cobb-Douglas production function:

$$Y_t = K_t^\theta (E_t L_t)^{1-\theta} \text{ and } \mu = (E_{t+1} - E_t)/E_t \quad (37)$$

where  $\theta$  is the capital share and  $E_t$  represents the level of technology.  $\mu$  is income per capita growth rate.  $L_t$  is the efficient unit of labor supply at period  $t$ .

Let  $1 - \hat{p}_i$  be the probability that an immigrant returns to her/his home country at the beginning of age  $i$ , given that she or he stays in the host country at the age  $i-1$  for  $i \geq 2$ . To simplify the notation, I assume that  $\hat{p}_1 = 1$ . Let  $N_t^{ij}$  be the number of agent of age  $i$  of type  $j$  at time  $t$ . Then,  $N_t^{im} = p_i \hat{p}_i N_t^{i-1,m}$  and  $N_t^{in} = p_i N_t^{i-1,n}$ .  $L_t$  is defined as follows:

$$L_t = \sum_{i=1}^{45} \sum_{j=n,m} H^{i,j} N_t^{ij} l_t^{i,j} \quad (38)$$

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<sup>19</sup>Alternatively, we can assume that immigrants will enjoy the same wage level in their home country as in the host country even if they return to their home countries. The two assumptions generate the same results.

I assume that all immigrants will arrive in the host country at age 1. Let  $\sigma_n$  be the growth rate of the number of natives of age 1 at the steady state and let  $\sigma_m$  be the growth rate of the number of immigrants of age 1 at the steady state. The total number of natives is  $\sum_{i=1}^{80} N_t^{in} = N_t^{1,n} \sum_{i=1}^{80} (1 + \sigma_n)^{-(i-1)} \times \prod_{q=1}^i p_q$ . The total number of immigrants is  $N_t^{1,m} \sum_{i=1}^{80} (1 + \sigma_m)^{-(i-1)} \times \prod_{q=1}^i p_q \hat{p}_q$ . This implies that the ratio of the total number of immigrants to the total number of natives is constant if and only if the growth rates of  $N_t^{1,n}$  and  $N_t^{1,m}$  are the same. Thus, for calculating the steady state, I assume that  $\sigma_n = \sigma_m = \sigma$ . I assume that the children of immigrants become native if their parents stay in the host country until the children attain adulthood. Then,  $N_t^{1,n}$  is determined by the fertility rates of both immigrants and natives and the return rate of immigrants. Let  $\eta_j^{i,j}$  be the age-specific fertility of type  $j$  at age  $i$ . The number of children born of type  $j$  parents of age  $i$  is  $N_t^{i,j} \eta_j^{i,j}$ . Let  $d$  be the infant-child mortality. Then the number of natives of age 1 at  $t + 20$ ,  $N_{t+20}^{1,n}$ , is determined as follows:

$$N_{t+20}^{1,n} = (1 - d) \times \left\{ \sum_{i=1}^{80} \eta^{i,m} N_t^{i,m} \prod_{x=1}^{20} \hat{p}_{i+x} + \sum_{i=1}^{80} \eta^{i,n} N_t^{i,n} \right\} \quad (39)$$

The percentage of immigrants to natives (PITN) at the steady state (see Appendix A6) becomes

$$\frac{\sum_{i=1}^{80} N_t^{i,m}}{\sum_{i=1}^{80} N_t^{i,n}} \times 100 = \frac{\frac{1}{1-d} - \sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta^{i,n} \times \prod_{q=1}^i p_q}{\sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta^{i,m} \times \prod_{q=1}^i p_q \hat{p}_q \prod_{x=1}^{20} \hat{p}_{i+x}} \quad (40)$$

$$\times \frac{\sum_{i=1}^{80} (1 + \sigma)^{-(i-1)} \times \prod_{q=1}^i p_q \hat{p}_q}{\sum_{i=1}^{80} (1 + \sigma)^{-(i-1)} \times \prod_{q=1}^i p_q} \times 100 \quad (41)$$

(40) says that the steady-state PITN is determined once the age 1 population growth rate,  $\sigma$ , the return rate of immigrants and the fertility rates of natives and immigrants are set. Conversely, we can choose  $\sigma$  so that the resulting PITN is consistent with the data<sup>20</sup>.

The capital stock at period  $t$  is the sum of the balance of individual savings and government savings. Let  $a_{t-1}$  be the balance of the government asset (or debt if it is negative) per capita at the end of period  $t - 1$ . Then, the total capital stock at period  $t$  is

$$K_t = \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1,i}^{i,j} s_{t-1}^{i,j} + a_{t-1} \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} \quad (42)$$

The (efficient unit) wage rate at time  $t$ ,  $w_t$  and the pre-tax interest rate at time  $t$ ,  $r_t$ , are determined as

$$w_t = (1 - \theta) K_t^\theta E_t^{1-\theta} L_t^{-\theta} \quad \text{and} \quad r_t = \theta K_t^{\theta-1} E_t^{1-\theta} L_t^{1-\theta} \quad (43)$$

<sup>20</sup>The RHS is an increasing function of  $\sigma$  at the parameter values that are consistent with the US. data.

Now, consider the initial balanced growth path where the capital to labor ratio (in efficient units) stays constant. Let  $w^*(1 + \mu)^t$  and  $r^*$  be the wage rate and interest rate at period  $t$  on the initial balanced growth path. Let  $s^{*i,j}(1 + \mu)^t$  and  $b^{*i,j}(1 + \mu)^t$  be the savings and the social security benefit of type  $j$  of age  $i$  at period  $t$  on the initial balanced growth path. Let  $a^*(1 + \mu)^t$  and  $g^{*i,j}(1 + \mu)^t$  be the government asset(debt) per capita and the publicly provided private goods for type  $j$  of age  $i$  at period  $t$  on the initial balanced growth path. It is possible that immigrants use more public services, for example, children's education. Thus, I put superscript  $j$ . Let  $w^*$  and  $r^*$  be the efficiency unit wage rate and the interest rate on the initial balanced growth path determined from (42) and (43). Then the government budget constraint on the initial balanced growth path is

$$(1 + \mu)^{t-1} \left\{ \begin{aligned} & \tau_{wt} w^*(1 + \mu) L_t + \tau_{rt} r^* \sum_{i=2}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s^{*i-1,j} + (1 + r^*) \sum_{i=2}^{80} \sum_{j=n,m} (1 - p_i) N_{t-1}^{i-1,j} s_{t-1}^{*i-1,j} \\ & - a^*(1 + \mu) \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} + (1 + r_t) a^* \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} - \sum_{i=1}^{80} \sum_{j=n,m} N_{t,i}^{i,j} g^{*i,j}(1 + \mu) \\ & - \sum_{i=46}^{80} \sum_{j=n,m} b^{*i,j}(1 + \mu) \times N_t^{i,j} - \kappa \sum_{i=46}^{80} \sum_{x=11}^{45} b^{*i,rm,x}(1 + \mu) \times N_t^{i,rm,x} \end{aligned} \right\} = 0 \quad (44)$$

where  $\tau_w^*$  and  $\tau_r^*$  are the wage tax rate and capital income tax rate at the initial balanced growth path.  $b^{*i,rm,x}$  and  $N_t^{i,rm,x}$  are, respectively, the social security benefit and the number of immigrants of age  $i$  who returned to the home country at age  $x$ , but who are still eligible to claim social security benefits.  $\kappa$  is the share of those returned immigrants who actually claim social security benefits. The US social security does not require the residence as long as the individual is eligible for benefits. I assume that an immigrant is eligible to receive social security benefits as long as he or she pays social security contributions for at least 10 years. The first term of (44) is the revenue from wage tax and the second term is the revenue from capital income tax. The third term is the revenue from inheritance tax. In my simulation, I assume that the government imposes a 100 percent inheritance tax. The fourth term is the revenue from issuing government bonds and the fifth term is the expenditure on the principal and interest of the bonds. The sixth and seventh terms are the expenditure on the publicly provided private goods and the expenditure on social security benefits of natives and immigrants who reside in the host country. The last term is the social security benefits of immigrants who returned to the home country and who are eligible to receive social security benefit.

When the government increases the PITN, the wage rate decreases and the interest rate

increases. To Pareto-improve welfare, I assume that the wage tax rate, interest tax rate and social security benefit are adjusted as in the theoretical analysis. More specifically, I assume that the government keeps the after-tax wage rate and interest rate at the same level as on the initial balance growth path. This implies that the wage tax rate  $\tau_{wt}$  and interest tax rate  $\tau_{rt}$  are set as follows:

$$w_t(1 - \tau_{wt}) = w^*(1 + \mu)^t \times (1 - \tau_w^*) \text{ and } r_t(1 - \tau_{rt}) = r^*(1 - \tau_r^*) \quad (45)$$

The social security benefit is adjusted as follows:

$$b_{t-1+i}^{i,j} = 12 \times RR \times AIME^j(t) \text{ and } AIME^j(t) = \frac{\sum_{i=1}^{45} (1 + \mu)^{45-i} w_{t-1+i} (1 - \tau_{wt}) l_{t-1+i}^{i,j} H^{i,j}}{(1 - \tau_w^*) \times 45 \times 12} \quad (46)$$

When the taxes and social security benefit formula are adjusted according to (45) and (46), the individual budget constraints is the same as on the initial balanced growth path<sup>21</sup>. Thus, consumption, labor supply and savings do not change. Then, there would be a surplus for the government budget even if the government were to spend the same amount of publicly provided private goods per person as at the initial balanced growth path, as analyzed in the previous section. The government can use this surplus to increase savings or to increase the level of publicly provided private goods. Let  $V$  be the distributional parameter that indicates what percentage of the budget surplus is put into savings. The government can keep increasing the government savings balance until the economy reaches the golden rule level or a modified golden rule level. As long as the MPK is greater or equal to the (modified) golden rule level, the balance of the government savings at period  $t$  is determined from the following equation:

$$(1 + \mu)^t a_t \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} = V \times SP_t$$

where  $SP_t$  is the government budget surplus at period  $t$ . If the MPK is equal to the (modified) golden rule level,  $V$  becomes 0.  $SP_t$  is defined as follows:

$$SP_t = \tau_{wt} w_t L_t + (1 + \mu)^{t-1} \left\{ \tau_{rt} r_t \sum_{i=1}^{80} \sum_{j=n,m} p_i N_{t-1}^{i-1,j} s^{*i-1,j} + (1 + r_t) \sum_{i=1}^{80} \sum_{j=n,m} (1 - p_i) N_{t-1}^{i-1,j} s^{*i-1,j} \right. \\ \left. (1 + r_t) \times a_{t-1} \sum_{i=1}^{80} \sum_{j=n,m} N_{t-1}^{i,j} - \sum_{i=1}^{80} \sum_{j=n,m} N_t^{i,j} \times g^{*,i,j} (1 + \mu) \right. \\ \left. - \sum_{i=46}^{80} \sum_{j=n,m} b^{*,i,j} (1 + \mu) \times N_t^{i,j} - \kappa \sum_{i=46}^{80} \sum_{x=11}^{45} b^{*,i,rm,x} (1 + \mu) \times N_t^{i,rm,x} \right\} \quad (47)$$

<sup>21</sup>Note that when  $w_t(1 - \tau_{wt}) = w_t^*(1 - \tau_w^*)$ ,  $AIME^j(t) = \frac{\sum_{i=1}^{45} (1 + \mu)^{45-i} w^*(1 + \mu)^{t-1+i} l_{t-1+i}^{i,j} H^{i,j}}{45 \times 12}$

The rest of the surplus that is available for increasing publicly provided private goods is  $(1 - V) \times SP_t$ . This is distributed equally to the entire population (including immigrants) at period  $t$ .<sup>22</sup> Then, after increasing the PITN, the amount of publicly provided private goods,  $g_t^i$ , becomes

$$g_t^i = g^{*i,j}(1 + \mu)^t + \frac{(1 - V) \times SP_t}{\sum_{i=1}^{80} (N_t^{i,n} + N_t^{i,m})} \quad (48)$$

For the (modified) golden rule, given the intergenerational discount rate for the modified golden rule, we say that the economy reaches the (modified) golden rule level if the marginal product of capital becomes equal to the sum of the growth rate of the efficiency unit of labor, the depreciation rate, and the inter generational discount rate for the modified golden rule. In the case of the golden rule, the intergenerational discount rate for the modified golden rule is set to zero.

## 4.2 Policy Experiments and Parameters Values for Simulation

### Policy Experiments

For the percentage of immigrants to natives (PITN), I calculate the data from the US census at 2000. US Census 2000 shows that the PITN above the age of 20 is 15.5 percent. Thus, I assume that the PITN at the initial balanced growth path is 15.5 percent. For the target PITN, I look at past US data. Historically, the PITN was 5 percent in 1970 and increased to 18.3 percent in 2010.<sup>23</sup> The PITN increased more than 10 percent within 40 years from 1970 to 2010, so I assume that a 10 percentage point increase in the PITN over 80 years is tolerable. Therefore, I set the target PITN at 25.5 percent. In the slow benchmark case, the PITN starts to increase from 15.5 percent and reaches the target PITN at the 80th year. After the 80th year, it remains on the same level. To see how accelerating the speed at which the PITN is increased affects my results, I also consider intermediate and fast cases,

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<sup>22</sup>The government could distribute this surplus more to the natives than to the immigrants. Thus, this assumption underestimates the welfare gain to the natives.

<sup>23</sup>Data in 1970 is from the US Census. Data in 2010 is from the CPS. After 2000, data on the foreign-born are available only through the American Community Survey and the CPS, not the Census. For calculating the PITN, one concern is the comparability of the CPS and the Census data. In 2000 when both the census and the CPS are available, the PITN in the Census is 12.42% and 11.54% in the CPS. Thus, the difference between the Census and CPS would seem to be very minor. This view is shared by the Census bureau. Schmidley and Robinson (2003) conclude that the difference of the estimates based on the Census data and CPS data are trivial.

in which the PITN hits the target PITN at the 62nd or 42nd year.<sup>24</sup> I assume that all immigrants arrive at age 1. The graph for the PITN over time is shown in Figure 1.<sup>25</sup>

### Fertility, Mortality and Return Rate of Immigrants

For the age-nativity specific fertility, I use the CPS 2000 June supplement, which the census bureau also uses for calculating on the age-nativity specific fertility. Figure A2 in the appendices shows the average of the total number of births by each woman's age and it shows that immigrant women have a greater total number of births than native women at all ages. From this figure, I calculate  $\eta^{i,n}$  and  $\eta^{i,m}$ , the age-specific birth rate for native and immigrant women. This is shown in Table A1. For the adult mortality profile,  $p_i$ , I take the values from Nishiyama and Smetter (2007). I set  $d$ , the sum of infant and child mortality, 1.7 percent, using the Vital Statistics of the US for 1993. For the return rate of immigrants to their home countries, I use the official census estimate that was conducted by Ahmed and Robinson (1994). They estimate that for the first 10 years, second 10 years and the third 10 years, the return rate of immigrants is 19 percent, 9 percent and 7 percent, respectively. For annual rate, those numbers correspond to 2.085 percent, 0.938 percent and 0.723 percent, respectively. Thus, I set  $\hat{p}_i = 0.9715$  for  $2 \leq i < 10$ ,  $\hat{p}_i = 0.99062$  for  $11 \leq i \leq 20$ ,  $\hat{p}_i = 0.9927$  for  $21 \leq i \leq 30$  and  $\hat{p}_i = 1$  for  $i \geq 31$ .

Once the age-nativity specific fertility rate, infant-child mortality and the initial PITN are set, then the annual population growth rate is calculated automatically according to equation (40) with the assumption that the PITN is at the steady state. With the estimated age-nativity specific fertility, the infant-child mortality and an initial PITN of 15.5 percent, the annual growth rate of the population of age 1 becomes 0.39 percent.<sup>26</sup> This implies that

<sup>24</sup>In the fast and intermediate cases, to allow smooth transition of the PITN, the PITN keeps increasing even after hitting the target PITN. See Figure 1.

<sup>25</sup>More specifically, each of the three cases is calculated as follows. Let  $f_{15.5}$  be the steady-state age1 immigrant-native ratio when the PITN is 15.5 percent. Define  $f_{25.5}$  in a similar fashion. Then, the age 1 immigrant-native ratio at period  $t$ ,  $f_t$ , in the slow case is  $f_t = f_{25.5}$  for all  $t$ . For the intermediate case,  $f_t = f_{25.5} + 0.1 \times f_{25.5} \times (\frac{t}{15})$  for  $1 \leq t \leq 15$ ,  $f_t = 1.1 \times f_{25.5} - 0.1 \times f_{25.5} \times (t-15)$  for  $16 \leq t \leq 30$  and  $f_t = f_{25.5}$  for  $31 \leq t$ . For the fast case,  $f_t = f_{25.5} + 0.4 \times f_{25.5} \times (\frac{t}{15})$  for  $1 \leq t \leq 15$ ,  $f_t = 1.4 \times f_{25.5} - 0.4 \times f_{25.5} \times (t-15)$  for  $16 \leq t \leq 30$  and  $f_t = f_{25.5}$  for  $31 \leq t$ .

Note that the steady-state age 1 immigrant-native ratio is defined in the first term of the RHS in equation (40). Thus, once the PITN is determined, the steady state age 1 immigrant-native ratio is calculated from equation (40).

<sup>26</sup>The annual CPS data on immigrants and natives from 1995 to 2010 shows that the median annual growth rate of the total population(sum of natives and immigrants) aged from 20 to 40 is 0.12% while the median

at the initial steady state, the government accepts immigrants such that the annual growth rate of immigrant of age 1 becomes 0.39 percent.

### Age-nativity Specific Government Expenditure

I assume that the age-specific government expenditure,  $g^{*i,j}$  is the same for natives and immigrants. Empirical studies show that there is no systematic difference in the use of public services by the two groups.<sup>27</sup> Thus, I assume that for  $1 \leq i \leq 24$ ,  $g^{*i,j} = g^y$ , for  $25 \leq i \leq 44$ ,  $g^{*i,j} = g_m$  and for  $45 \leq i \leq 80$ ,  $g^{*i,j} = g_o$ . I assume that  $g^y, g^m$  and  $g^o$  are 24.5%, 13.4% and 23.2 % of GDP per capita at the initial balanced growth path following Storesletten (1995) and Auerbach, Kotlikoff Hagememann and Nicoletti (1989). On the other hand, to check the robustness of my results, I also assume that immigrants consume 20 percent more publicly provided private goods than natives in the robustness checks.

### Utility Function, Production Function and Human Capital Profile

A precise estimate for the coefficient for relative risk aversion,  $\gamma$ , has not yet been found in the literature. Auerbach and Kotlikoff (1986) and Storesletten (2000) assumed that  $\gamma$  is 4. Nishiyama and Smetter (2007) set  $\gamma$  equal to 2. I assume that  $\gamma = 3$  and check the robustness with  $\gamma = 2$  and  $\gamma = 4$ . For the time preference rate,  $\beta$ , following Hurd (1989) and Storesletten (2000), I assume that  $\beta = 1.011$ . Higher  $\beta$  implies higher savings and a higher capital to output ratio. To check the sensitivity of my results, I also calculate with  $\beta = 1$  and  $\beta = 0.99$ . I assume that the leisure share in the utility function,  $\zeta$ , is 0.33.

I assume that the depreciation rate of capital,  $\delta$ , is equal to 0.047. For the capital share in the production function,  $\theta$ , I set  $\theta = 0.4$ . For technological progress, I assume that income per capita growth rate,  $\mu$ , is 0.015.

For the human capital profile of natives,  $H_i^s$ , I take the value from Auerbach and Kotlikoff (1986). The growth rate of immigrants aged from 20 to 40 is 1.85%. In the theoretical model, the model assumes that the age 1 population growth rate of immigrants and natives are the same. Thus, the theoretically predicted growth rate of age 1 population, 0.39%, is between the growth rate of immigrants aged 20-40 and the growth rate of the total population aged 20-40.

<sup>27</sup>Borjas and Hilton (1996) show that immigrants have a higher participation rate in welfare programs than natives. Fix, Passel, and Zimmermann (1996) show that these differences are explained by the higher participation rate in welfare programs among refugees and retired immigrants and that there is no differences among labor immigrants. Thus, for the theoretical part, I assume that  $g^i$  is independent of the place of birth. For the computational part, I relax this assumption for the robustness checks.



likoff.

$$H^{i,n} = \exp(4.47 + 0.033 \times i - 0.00067 \times i^2) \quad \text{for } 1 \leq i \leq 45 \quad (49)$$

$$H^{i,n} = 0 \quad \text{for } 46 \leq i \quad (50)$$

For the human capital profile of the immigrants, Storesletten (1995) showed that immigrants' earnings are, on average, 15.7 percent lower than those of natives.<sup>28</sup> Similarly, using the CPS June 2000 supplement, my calculations show that immigrants' earnings are 10 percent lower than those of natives. Using these estimates as a basis, I assume that the efficient unit of human capital of immigrants is 84.3 percent of that of natives and that  $H^{i,m} = 0.843 \times H_i^n$  in the benchmark calculation. To examine the robustness of my results, I change the level of human capital from 84.3 percent to 89.3 percent, 79.3 percent or 74.3 percent and re-check the results.

### Taxes and Government Debt

For capital income tax, I take the value from Nishiyama and Smetter (2007) and assume that  $\tau_k = 0.28$ . For the level of social security benefit, a higher replacement rate means that greater intergenerational redistribution of income, which leads to greater welfare gain as a result of increasing the immigrant population. Following Auerbach and Kotlikoff, in the benchmark case I set the replacement 0.6 and check the robustness of my results by varying it from 0.6 to 0.55 and 0.5.

As for the initial level of government debt or assets, different authors set different levels. Storesletten considered only government debt and assumed that the initial level is 50 percent of the initial GDP. With his estimate of the initial capital to output ratio being 2.4, his assumption implies that government debt is about 20 percent of private capital. Nishiyama and Smetter considered not only government debt, but also government assets by using the BEA information on the government's fixed capital. They assumed that at the initial steady state, the government has positive net assets of 10 percent of the total private capital. This naturally leads to a higher capital to output ratio at the initial balanced growth path. Herein, as a benchmark case, I assume that initial level of government debt or assets is 0 percent of the private capital and I experiment with the assumption that the government debt is 10 percent or minus 10 percent of the private capital.

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<sup>28</sup>Figure 2.2 of Storesletten (1995) shows that at age 20, 25, 30, 35, 40, 45, the wage rate of immigrants is lower than that of natives by 15%, 20%, 17.8%, 16.4%, 12% and 13% respectively. By averaging those rates, I obtained a working value of 15.7%.

### 4.3 Results

Figures A3, A4 and A5 in the appendices show the balance of assets, consumption and leisure for the life cycle of an individual at the initial balanced growth path in the benchmark analysis. At the age of 46, the consumption of leisure becomes 1 due to mandatory retirement. At the initial balanced growth path, the capital to output ratio is 2.98, which is higher than the value used in Storesletten (2.4) and Nishiyama and Smetter (2.7), but lower than the value used in the standard business cycle research, which is 3.2 (Cooly and Prescott(1995)). To check how my results are affected by the capital to output ratio, I change the time preference rate and examine how the results change for different values of the time preference rate in the robustness checks.

Table 1 shows the parameter values and the welfare effect of increasing the PITN for different values of  $V$  in equation (47), which is the share of the government surplus that is put into savings. As mentioned above, the initial PITN is set at 15.5 percent and the target INR at 25.5 percent. With respect to rate at which immigrants are accepted, I consider three cases in which it takes 80, 62 and 42 years for the PITN to reach the target PITN. Column (3) of Table 1 shows the share of the government surplus that is put into savings. Columns (4)-(9) are values that are calculated within the simulation. Column (4) shows how many years it takes for the economy to reach the (modified) golden rule level. When it does not reach the (modified) golden rule within 300 years, this is indicated by \* or \*\*. \* indicates that the capital-to-labor ratio (efficient unit) at the 300th year is higher than at the initial balanced growth path, that it is increasing at the 300th year, but that it does not reach the (modified) golden rule level at the 300th year. \*\* indicates that the capital-to-labor ratio at the 300th year is lower than at the initial balanced growth path. For example, when  $V = 100\%$ , it takes 112 years to reach the golden rule level in the slow acceptance case(row (16)). Column (5) shows how much the capital stock per efficient unit of labor increases at the golden rule compared with the level at the initial balanced growth path. When  $V = 100\%$ , it shows that the capital stock per efficient unit of labor increases by 102 percent. Column (6) shows how much the publicly provided private goods will increase compared with the level at the initial balanced growth path. At  $V = 100\%$ , column (6) shows that the publicly provided private goods will increase by about 36 percent. To calculate columns (5) and (6), I evaluate at the year at which the economy reached the golden rule level if it reaches that level within 300 years, and at the 300th year otherwise. Column (7) shows the percentage by which the utility, measured by the expenditure function, of the cohort that is

born when the economy reaches the golden rule level increases compared with the utility of the same cohort if it were on the initial balanced growth path. When the economy does not reach the golden rule level, I calculate the utility of the cohort that is born at the 300th year. In the expenditure function, the price vector on the initial balanced growth path is used for evaluating the utility. Note that all welfare gain is distributed through an increase in publicly provided private goods. Column (8) shows the share of the present discounted value (PDV) of the increased publicly provided private goods in the initial GDP. For example, in row (16), the PDV of the increased publicly provided private goods is 12 percent of the initial GDP. For discounting the increased publicly provided private goods in future periods, I apply a 5 percent discount rate instead of the equilibrium interest rate, so that I can eliminate the effect of the discount rate when comparing different cases. Column (9) measures the welfare gain of natives and their descendants, not that of immigrants or their descendants. It measures how much the utility of natives and their descendants, not including immigrants and their descendants, is Pareto-improved by accepting more immigrants compared with the utility level at the initial balanced growth path. It evaluates the utility by using the expenditure function. I apply the price vector at the initial balanced growth path for the expenditure function and the equivalent variation to measure the difference of the levels of utility in two cases. To discount the welfare gain of future cohorts, I again use a 5 percent discount rate. Note that in all cases in Table 1, I assume that the government does not discriminate between immigrants and natives with respect to publicly provided private goods. Both natives and immigrant consume publicly provided private goods equally. Column (9) in row (16) shows that the PDV of the Pareto-improving welfare gain of natives and their descendants comprises 11 percent of the initial GDP.

In rows (2)-(3), (5)-(6), and (8)-(9)..., I shorten the years needed to reach the target PITN and increase the speed with which the PITN increases. When the number of years needed to reach the target PITN is reduced to 42, instead of 80, and  $V=100$ , the PDV of the welfare gain of increasing the PITN comprises 13 percent of the initial GDP (row (18)).

Table 1 shows that the number of years taken to reach the golden rule level decreases as  $V$  increases, because the government saves more for future cohorts. On the other hand, the PDV of the increased utility, measured as the share of the initial GDP, increases as  $V$  decreases as long as the  $V$  is greater than or equal to 50%. The PDV of the increased utility, measured as the share of the initial GDP, is highest when  $V=50\%$ . In this case, the quantified Pareto improvement ranges from 21% to 26% of the initial GDP.

In Table 1, I set the intergenerational discount rate for the modified golden rule at 0% and set the target level of the capital stock at the golden rule level. However, the targeting capital stock at the golden rule level does not necessarily maximize the PDV of the welfare gain. In Table A2, I examine the effect on welfare of increasing the PITN for different target levels of capital stock by changing the value of the intergenerational discount rate for the modified golden rule when  $V=100\%$ . Table A2 shows that the welfare gain is maximized when the intergenerational discount rate is set at 3 percent.

In Table 2, which I consider as the representative case of my simulation, I recalculate the all rows of Table 1 by setting the intergenerational discount rate for the modified golden rule at 3 percent. In Table 2, the Pareto-improving welfare gain of increasing the PITN is more than 20 percent of the initial GDP and the capital stock per efficient unit of labor increases by 18 percent as long as  $V$  is greater than or equal to 50 percent.

Figure 2 and Figure A6 show the marginal product of capital and the capital to output ratio over time for different values of  $V$  when the target capital stock is the modified golden rule level (3% intergenerational discount rate). The marginal product of capital increases initially, due to the acceptance of more immigrants. However, as the government savings balance increases, the capital stock per efficient unit of labor starts to increase and the marginal product of capital continues to decrease until the economy reaches the golden rule level. The capital to output ratio also displays a similar pattern.

Figure 3 compares the utility level of all cohorts on the initial balanced growth path with the utility level of all cohorts for different values of the share for the government savings ( $V$ ) when the target capital stock is set at the golden rule level. For example, when  $V=100\%$ , all the surplus is put into savings until the economy reaches the golden rule level and the surplus is distributed to individuals only after the economy reaches the golden rule level. This implies that the utility of the 65th cohort, which dies at the 65th year, starts to experience higher utility than at the initial balanced growth path.

In all cases considered in Tables 1,2,3 and 4, the results of simulations show that all cohorts are Pareto-improved; this confirms my theoretical results.<sup>29</sup>

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<sup>29</sup>The results of this simulation differ from those of some of the previous studies. For example, Kotlikoff et.al reports that the welfare gain of doubling immigrants is very small. Lee and Miller argues that the fiscal impact of accepting additional 100,000 immigrants is very small. Several factors in those studies generate different results. Kotlikoff et.al analyze immigration policy in an open economy setting, whereas I use a closed economy setting. In an open economy setting, the effect of additional government savings is offset by the mobility of capital. In my simulation, the government can use the budget surplus, which is obtained by

Figure 4 shows the importance of government savings balance at the new equilibrium path. It calculates the ratio between the interest income from the government savings balance and the social security benefit payment in each period. When the economy reaches the golden rule level, the interest income from the government savings balance comprises 70 percent of the social security benefit payments. Thus, at the new equilibrium path, the interest income from the government savings balance is contributing substantial amount.

#### 4.4 Robustness Checks

Table 3 and 4 present results of robustness checks of Table 2. Rows (1)-(6) in Table 3 check whether the results in Table 2 are sensitive to the initial government debt (asset) level. As I argued above, different authors assume different levels for the government debt (asset) at the initial balanced growth path. In rows (1)-(3) in Table 5, I set the initial government debt to 10 % of the private capital instead of 0 %. In rows (4)-(6), I assume that the initial government debt level is -10% of the private capital.

Rows (7)-(12) in Table 3 check whether the results in Table 2 are sensitive to the replacement rate. The theoretical analysis implies that higher intergenerational redistribution will result in the acceptance of more immigrants yielding a higher welfare gain. Thus, it is predicted that as the replacement rate decreases, so does the welfare gain of accepting more immigrants. Rows (7)-(12) confirms this prediction. Decreasing the replacement rate 10 percentage points decreases the welfare gain by 7 percentage point in terms of the percentage of the initial GDP.

Rows (13)-(21) in Table 3 conduct sensitivity checks on the immigrants' level of earnings. Following Storesletten (1995), I set the immigrant wage rate at 84.3 percent of that of natives. In my calculation using the CPS 2000 June supplement, I found that immigrants' earning are 91 percent of natives' earnings. Rows (13)-(15) assume that the wage rate of immigrants is 89.3 percent of that of natives, rather than 84.3 percent. Rows (16)-(18) assume that the wage rate of immigrants is 79.3 percent of that of natives. Rows (19)-(21) assume that the wage rate of the immigrants is 75 percent of that of natives. The results in rows (19)-(21) show that if immigrants earn 25 percent lower than natives, the welfare gain 

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accepting more immigrants, for savings and the amount of savings affects the over all welfare. Lee and Miller consider a much smaller increase in the number of immigrants than I do. On the other hand, my simulation results are consistent with Auerbach Oreopoulos (2000). They find that if the initial fiscal imbalance is not adjusted, then the welfare loss of halving the number of new immigrants is substantial. This is consistent with my theoretical and simulation results.

of accepting more immigrant is 17 percent of the initial GDP, instead of 23 percent of the initial GDP.

Rows (22)-(27) in Table 3 conduct sensitivity checks on the consumption by immigrants of publicly provided private goods. In Table 1 and 2, I assume that immigrants and native consume publicly provided private goods equally as the previous empirical studies indicate. In rows (22)-(24), I assume that young immigrants consume 20 percent more publicly provided private goods than young natives. The PDV of the welfare gain comprises 17 percent of the initial GDP instead of 23 percent. Rows (25)-(27) assume that immigrants of all ages consume 20 percent more publicly provided private goods than natives. In this case, the PDV of the welfare gain comprises 15 percent of the initial GDP.

In Table 4, I conduct robustness checks by changing the values of parameters of the utility function and the return rate of immigrants. Rows (1)-(6) of Table 4 examine the sensitivity of the results regarding the values of the parameters for constant relative risk aversion (CRRA). In Table 1-3, I assume that the CRRA is equal to 3. Auerbach and Kotlikoff, and Storesletten assume that the CRRA=4, while Nishiyama and Smetter assume that the CRRA is 2. Rows (1)-(3) assume that the CRRA is equal to 4 and rows (5)-(6) assume that it is 2. The results presented on rows (1)-(6) show that the results presented in Table 2 is not sensitive to the value of CRRA. Rows (7)-(12) examine the sensitivity of the results to the time preference rate. Although it is quite common to assume that the time preferences rate is greater than 1, readers might think that the results presented in Table 1 and 2 can be sensitive with respect to the assumption that the time preference rate is greater than 1. Theoretically, lowering the time preference rate will result in lowering savings and will decrease the capital to labor ratio at the initial balanced growth path. Since increasing the number of immigrants will increase the capital stock and the production function exhibits diminishing marginal product of capital, lowering the initial capital stock as a result of assuming a lower time preference rate will increase the welfare gain of increasing the PITN. Rows (7)-(12) confirm this theoretical prediction, but they show that the magnitude of those changes is very small. For example, changing the time preference rate from 1.011 to 0.99 increase the PDV of the welfare gain by only 1 percentage point.

Rows (13)-(15) in Table 4 conduct robustness checks by assuming that the return rate of immigrants is equal to 0. Rows (13)-(15) show that the results presented in Table 2 do not change so much and that they are robust regarding the return rate of immigrants.

## 4.5 Summary of the Simulation Results

The simulation results presented in Tables 1-4 can be summarized as follows. In the case of slow acceptance of more immigrants, in which the PITN increases from 15.5 percent and reaches 25.5 percent at the 80th year, the PDV of the increased utility of the natives and their descendants, measured by the expenditure function, is 23 percent of the initial GDP when the target capital stock is the modified golden rule level (3 % intergenerational discount rate). The economy reaches the modified golden rule level at 65th year and the capital stock per efficient unit of labor increases by 18 percent. In the fast acceptance case, in which the PITN reaches 25.5 percent at the 42th year, the PDV of the welfare gain comprises 28 percent of the initial GDP and the economy reaches the modified golden rule level at 59th year.

For robustness checks, I conduct sensitivity checks for changing the values of the following parameters: initial government debt (asset) level, the replacement rate, level of earnings of immigrants, immigrants' consumption of publicly provided private goods, the CRRA, the time preference rate and probability of the return to home country by immigrants. The sensitivity checks show that the results are quite robust.

## 5 Conclusion

I have examined, both theoretically and quantitatively, the effect on welfare of increasing the PITN in the presence of a PYGO social security system. The results for both the theoretical quantitative analysis show that the welfare gain of accepting more immigrants is robust and non-trivial. The PDV of the welfare gain of increasing the PITN from 15.5 to 25.5 percent amounts to about 20 percent of the initial GDP. In the shortest case, the economy reaches the golden rule level at the 112th year in a Pareto-improving way. The analysis suggests that accepting more immigrants can be an important tool for policy makers when addressing the economic problems caused by a PYGO social security system.

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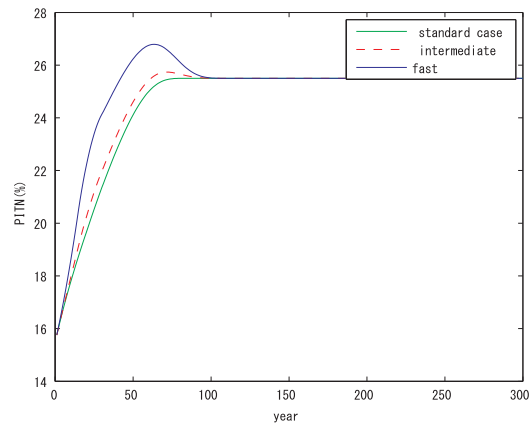


Figure 1: PITN over time. The initial PITN is set at 15.5% and the target PITN is set at 25.5%.

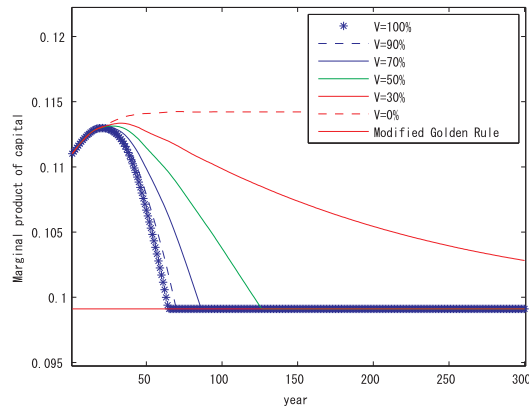


Figure 2: The marginal product of capital over time for different values of the share of the surplus used for government savings ( $V$ ). The target capital stock is the modified golden rule level with 3 % intergenerational discount rate.

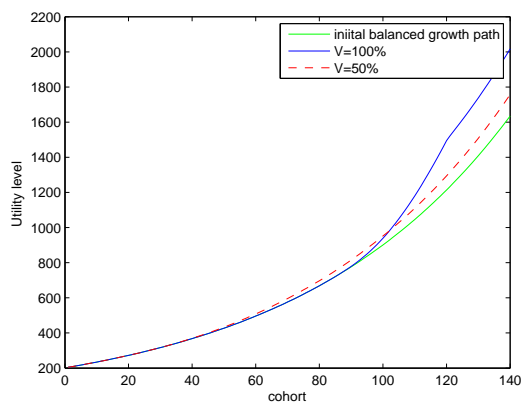


Figure 3: Utility level of different cohorts for different values of the share of the surplus used for government savings ( $V$ ). The target capital stock level is the modified golden rule level with 3% intergenerational discount rate for the modified golden rule.

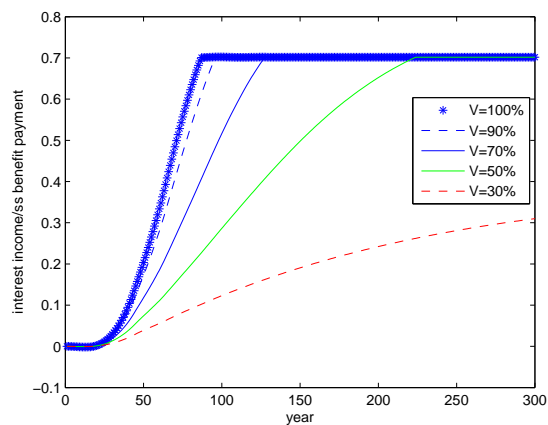


Figure 4: The ratio between the interest income from the government savings balance and the social security benefit payment in each period. It is assumed that the PITN reaches the target PITN at 80th year. The inter-generational discount rate for the modified golden rule is set at 3%.

Table 1  
The effect of increasing the PITN  
(The target capital stock is the golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	share of the surplus for the gov. savings (V)	year reaching the golden rule	% increase of capital stock per efficient unit labor at the golden rule	% change of publicly provided private goods per capita at the golden rule	% change of welfare of cohort born at the golden rule	share of the sum of the PDV of increased publicly provided private goods in the initial GDP	share of the sum of PDV of welfare gain of all natives and their descendants in the initial GDP
share of the surplus for the gov. savings =0%								
1	80	0%	300**	-4.37%	5.00%	0.53%	15.57%	17.72%
2	62	0%	300**	-4.37%	5.00%	0.53%	16.74%	18.94%
3	42	0%	300**	-4.37%	5.00%	0.53%	20.20%	22.47%
share of the surplus for the gov. savings =30%								
4	80	30%	300*	11.89%	14.69%	1.41%	17.86%	19.59%
5	62	30%	300*	12.00%	14.76%	1.42%	19.20%	20.94%
6	42	30%	300*	73.43%	30.82%	2.88%	25.32%	26.09%
share of the surplus for the gov. savings =50%								
7	80	50%	300*	71.21%	30.27%	2.79%	19.66%	20.79%
8	62	50%	300*	71.82%	30.42%	2.81%	21.10%	22.18%
9	42	50%	300*	73.43%	30.82%	2.88%	25.32%	26.09%
share of the surplus for the gov. savings =70%								
10	80	70%	184	102.43%	35.86%	3.77%	18.48%	18.92%
11	62	70%	182	102.43%	35.86%	3.77%	19.75%	20.09%
12	42	70%	119	50.99%	30.14%	3.17%	25.87%	25.56%
share of the surplus for the gov. savings =90%								
13	80	90%	127	102.43%	35.86%	3.77%	14.52%	13.75%
14	62	90%	125	102.43%	35.86%	3.77%	15.47%	14.56%
15	42	90%	120	102.43%	35.86%	3.77%	18.21%	16.80%
share of the surplus for the gov. savings =100%								
16	80	100%	112	102.43%	35.86%	3.77%	12.23%	10.69%
17	62	100%	110	102.43%	35.86%	3.77%	13.04%	11.33%
18	42	100%	106	102.43%	35.86%	3.77%	15.38%	13.14%

Notes

1. In all rows, the initial PITN is 15.5% and target PITN is 25.5%. The replacement rate is 60 %, CRRA=3 and the time preference rate is 1.011. The equilibrium capital to output ratio on the initial balanced growth path is 2.98.
2. In all rows, wage rate of immigrants is 84.3 % of that of natives.
3. \* indicates that the capital stock per efficient unit labor does not reach the golden rule level within 300 years. Its value at the 300th year is higher than at the initial balanced growth path and keeps increasing at the 300th year. The percent change of the capital stock per efficient unit of labor is evaluated at the 300th year.
4. \*\* indicates that capital stock per efficient unit labor does not reach the golden rule level within 300 years and the capital stock per efficient unit labor at the 300th year is lower than at the initial balanced growth path. The percent change of the capital stock per efficient unit of labor is evaluated at 300th year.

Table 2  
The effect of increasing the PITN for different values of V  
(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	share of the surplus for the gov. savings (V)	year reaching the golden rule	% increase of the capital stock per efficient unit labor at the golden rule	% change of publicly provided private goods per capita at the golden rule	% change of welfare of cohort born at the golden rule	share of the PV of increased publicly provided private goods in the initial GDP	share of the PDV of welfare gain of all natives and their descendants in the initial GDP
share of the surplus for the gov. savings =0%								
1	80	0%	300**	-4.37%	5.00%	0.53%	15.57%	17.72%
2	62	0%	300**	-4.37%	5.00%	0.53%	16.74%	18.94%
3	42	0%	300**	-4.37%	5.00%	0.53%	20.20%	22.47%
share of the surplus for the gov. savings =30%								
4	80	30%	300*	11.89%	14.69%	1.41%	17.86%	19.59%
5	62	30%	300*	12.00%	14.76%	1.42%	19.20%	20.94%
6	42	30%	300*	12.32%	14.96%	1.45%	23.18%	24.80%
share of the surplus for the gov. savings =50%								
7	80	50%	127	18.19%	18.51%	1.95%	19.86%	21.06%
8	62	50%	124	18.19%	18.51%	1.95%	21.31%	22.47%
9	42	50%	116	18.19%	18.51%	1.95%	25.58%	26.45%
share of the surplus for the gov. savings =70%								
10	80	70%	87	18.19%	18.51%	1.95%	21.31%	22.11%
11	62	70%	78	18.19%	18.51%	1.95%	27.15%	27.49%
12	42	70%	78	18.19%	18.51%	1.95%	27.15%	27.49%
share of the surplus for the gov. savings =90%								
13	80	90%	70	18.19%	18.51%	1.95%	22.32%	22.85%
14	62	90%	68	18.19%	18.51%	1.95%	23.81%	24.24%
15	42	90%	63	18.19%	18.51%	1.95%	28.14%	28.13%
share of the surplus for the gov. savings =100%								
16	80	100%	65	18.19%	18.51%	1.95%	22.71%	23.14%
17	62	100%	63	18.19%	18.51%	1.95%	24.19%	24.51%
18	42	100%	59	18.19%	18.51%	1.95%	28.51%	28.37%

Notes

1. In all rows, the initial PITN is 15.5% and target PITN is 25.5%. The replacement rate is 60 %, CRRA=3 and the time preference rate is 1.011. The equilibrium capital to output ratio on the initial balanced growth path is 2.98.
2. In all rows, wage rate of immigrants is 84.3 % of that of natives.
3. The intergenerational discount rate for the modified golden rule is 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the intergenerational discount rate for the modified golden rule.
3. \* indicates that the capital stock per efficient unit labor does not reach the modified golden rule level within 300 years. Its value at the 300th year is higher than at the initial balanced growth path and keeps increasing at the 300th year. The percent change of capital stock per efficient unit of labor is evaluated at the 300th year.
4. \*\* indicates that the capital stock per efficient unit labor does not reach the modified golden rule level within 300 years and the capital stock per efficient unit labor at the 300th year is lower than at the initial balanced growth path. The percent change of the capital stock per efficient unit of labor is evaluated at the 300th year.

Table 3  
Robustness checks  
The role of the initial government debt level, the replacement rate, immigrants earnings and the use of public services by immigrants

(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	capital to output ratio at the initial balanced growth path	years taken to reach the modified golden rule level	% increase of capital stock per efficient unit labor at the new balanced growth path	% change of publicly provided goods per capita at the new balanced growth path	% change of welfare of the cohort born at the new balanced growth path	share of the sum of PDV of increased publicly provided private goods in the initial GDP	share of the sum of the PDV of welfare gain of all natives and their descendants in initial GDP
initial government debt ratio (% of private capital)= 10 %								
1	80	2.91	63	22.39%	21.87%	2.29%	28.48%	27.22%
2	62	2.91	61	22.39%	21.87%	2.29%	30.31%	28.82%
3	42	2.91	57	22.39%	21.87%	2.29%	35.66%	33.30%
initial government debt ratio(% of private capital)= -10 %								
4	80	3.04	67	14.71%	15.75%	1.67%	18.30%	19.68%
5	62	3.04	65	14.71%	15.75%	1.67%	19.51%	20.87%
6	42	3.04	61	14.71%	15.75%	1.67%	23.01%	24.16%
replacement rate =0.55								
7	80	3.03	68	14.87%	15.79%	1.67%	18.05%	19.32%
8	62	3.03	66	14.87%	15.79%	1.67%	19.24%	20.48%
9	42	3.03	61	14.87%	15.79%	1.67%	22.67%	23.70%
replacement rate =0.5								
10	80	3.1	71	11.51%	13.07%	1.39%	13.68%	15.38%
11	62	3.1	69	11.51%	13.07%	1.39%	14.572%	16.30%
12	42	3.1	65	11.51%	13.07%	1.39%	17.159%	18.84%
level of human capital of immigrants is 89.3 % of that of natives								
13	80	2.98	62	18.20%	18.96%	2.01%	25.22%	25.93%
14	62	2.98	60	18.20%	18.96%	2.01%	26.871%	27.48%
15	42	2.98	56	18.20%	18.96%	2.01%	31.702%	31.86%
level of human capital of immigrants is 79.3 % of that of natives								
16	80	2.98	69	18.18%	18.05%	1.89%	20.09%	20.24%
17	62	2.98	67	18.18%	18.05%	1.89%	21.392%	21.43%
18	42	2.98	62	18.18%	18.05%	1.89%	25.165%	24.74%
level of human capital of immigrants is 74.3 % of that of natives								
19	80	2.98	73	18.18%	17.58%	1.83%	17.32%	17.20%
20	62	2.98	71	18.18%	17.58%	1.83%	18.430%	18.20%
21	42	2.98	67	18.18%	17.58%	1.83%	21.629%	20.96%
young adult immigrants consume publicly provided private goods 20% higher than young adult natives								
22	80	2.97	74	18.46%	17.70%	1.90%	17.27%	17.01%
23	62	2.97	72	18.46%	17.70%	1.90%	18.343%	17.96%
24	42	2.97	68	18.46%	17.70%	1.90%	21.409%	20.56%
immigrants of all ages consume publicly provided private goods 20% higher than natives								
25	80	2.96	77	18.84%	16.89%	1.86%	15.71%	15.25%
26	62	2.96	75	18.84%	16.89%	1.86%	16.674%	16.09%
27	42	2.96	71	18.84%	16.89%	1.86%	19.410%	18.36%

Notes

- In all rows,  $V$  is 100 percent and the inter-generational social discount factor for the modified golden rule is set at 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.
- In all rows, the initial PITN is 15.5% and the target PITN is 25.5%.  $CRRA=3$  and the time preference
- The wage rate of immigrants is 84.3 % of that of natives in all rows except in rows (13) to (18).
- Immigrants consume the same amount of publicly provided private goods as natives in rows (22) to (27).
- The replacement is 0.6 in all rows except rows (7) to (12).

Table 4  
Robustness checks (2)

The role of the CRRA, the time preference rate and the return rate of immigrant

(The target capital stock is the modified golden rule level)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target INR	capital to output ratio at the initial balanced growth path	years taken to reach the modified golden rule level	% increase of the capital stock per efficient unit labor at the new balanced growth path	%change of publicly provided goods per capita at the new balanced growth path	% change of welfare of cohort born at the new balanced growth path	share of the sum of the PDV of increased publicly provided private goods in the initial GDP	share of sum of the PDV of welfare gain of all natives and their descendants in initial GDP
CRRA=4								
1	80	2.634	74	41.87%	33.40%	3.28%	32.25%	22.18%
2	62	2.634	72	41.87%	33.40%	3.28%	34.04%	23.26%
3	42	2.634	69	41.87%	33.40%	3.28%	39.10%	26.15%
CRRA=2								
4	80	3.44	63	11.52%	12.70%	1.47%	16.26%	25.82%
5	62	3.44	61	11.52%	12.70%	1.47%	17.52%	27.70%
6	42	3.44	56	11.52%	12.70%	1.47%	21.32%	33.23%
the time preference rate=1								
7	80	2.67	72	39.41%	32.00%	3.17%	32.35%	23.15%
8	62	2.67	71	39.41%	32.00%	3.17%	34.184%	24.31%
9	42	2.67	67	39.41%	32.00%	3.17%	39.396%	27.43%
the time preference rate=0.99								
10	80	2.42	74	61.43%	46.18%	4.38%	43.96%	23.52%
11	62	2.42	73	61.43%	46.18%	4.38%	46.130%	24.51%
12	42	2.42	70	61.43%	46.18%	4.38%	52.171%	27.09%
the retrun rate of immigrant is 0%								
13	80	2.97	63	17.04%	17.86%	1.87%	24.00%	24.25%
14	62	2.97	61	17.04%	17.86%	1.87%	25.56%	25.68%
15	42	2.97	61	14.87%	15.79%	1.67%	22.67%	23.70%

Notes

- In all rows, V is 100 percent. The parameter values of the wage rate of immigrants, the replacement rate, and the consumption of publicly provided private goods by immigrants are the same as in Table 2.
- In all rows, the initial PITN is 15.5% and the target PITN is 25.5%.
- The inter-generational discount rate for the modified golden rule is 3% except in rows (4) to (6). In rows (4) to (6), the economy's capital stock is already above the modified golden rule level with a 3% inter-generational discount rate. In rows (4) to (6), I set the inter-generational discount rate at 2% instead of 3%. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.

Appendices (The following appendices are put on the Journal's webpage as supplement material, but not included in the main text. )

Appendix A

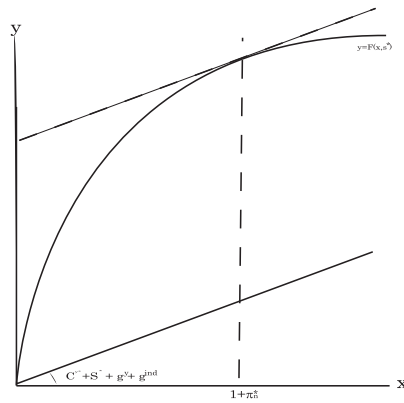


Figure A1- The one plus population growth rate and the resources used for one old person. The curve through the origin is  $y = F(x, s^*)$  and the straight line is  $y = (c^{y*} + s^* + g^y + g^{ind}) \times x$  where  $x$  is one plus the population growth rate. The vertical distance of the straight line through the origin measures the total resources used for young divided the number of old people. The difference between the curved line and the straight line measures the resource used per one old person at  $x$ . If  $x$  is lower than  $1 + \pi_n^*$ , increasing  $x$  will increase the resources available per one old person without decreasing the resources used for the young.



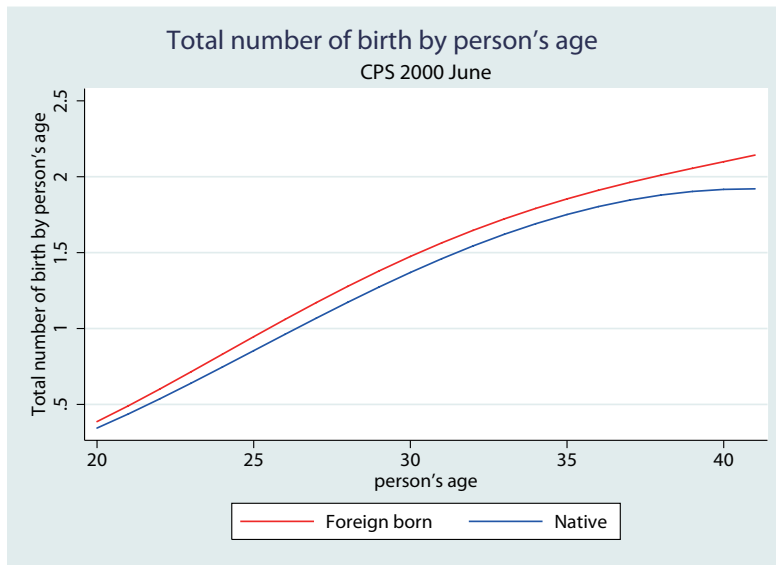


Figure A2–Total number of births by each age. The data source is the CPS 2000 supplement. The total number of births by each age is regressed on a sixth-order polynomial function of age separately for natives and immigrants. The predicted values are plotted.

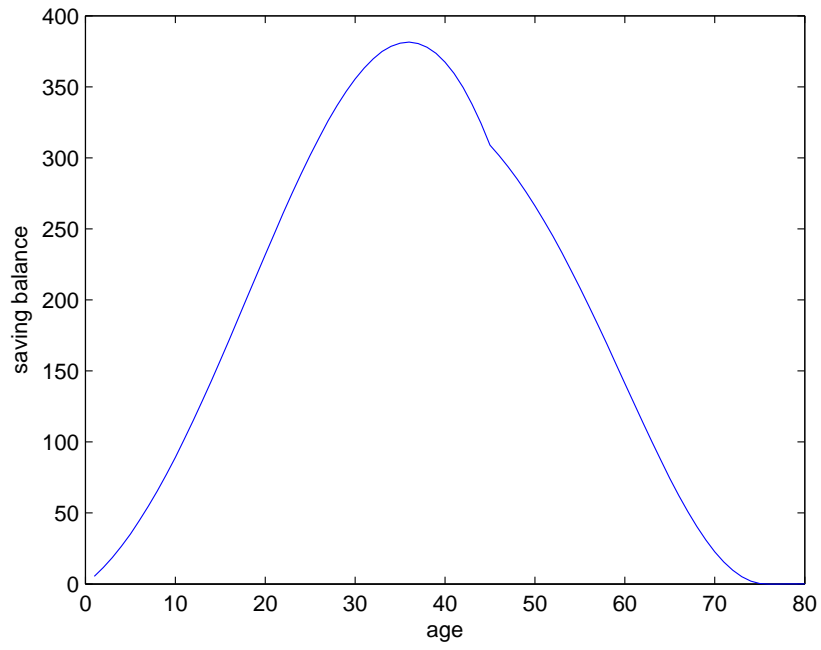


Figure A3–Balance of asset over the life cycle at the initial balanced growth path

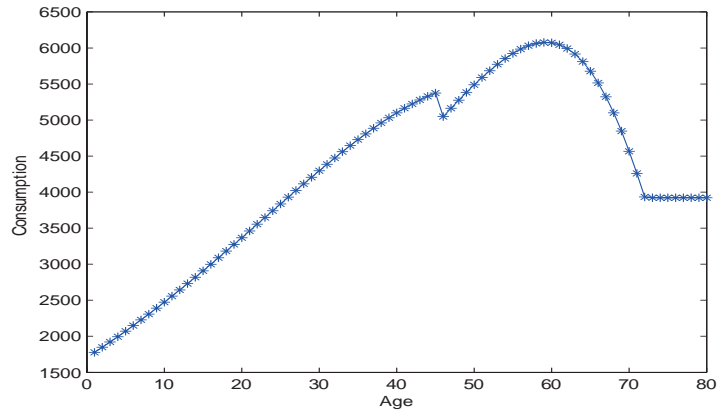


Figure A4–Life cycle consumption path at the initial balanced growth path.

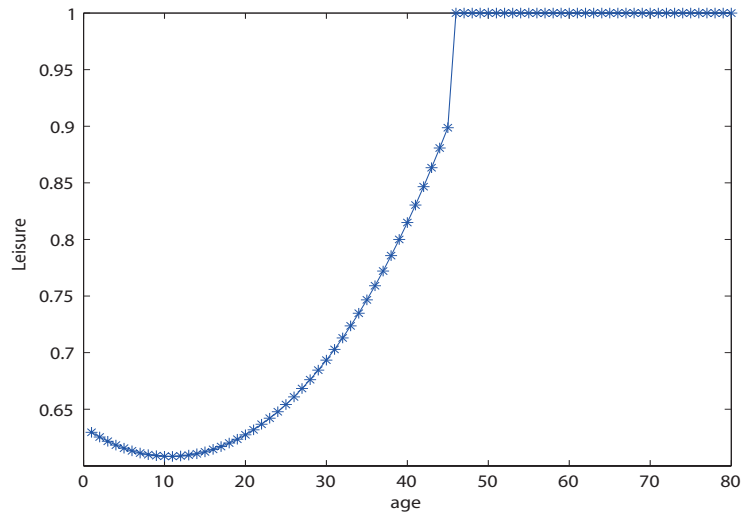


Figure A5–Life cycle consumption of leisure at the initial balanced growth path.

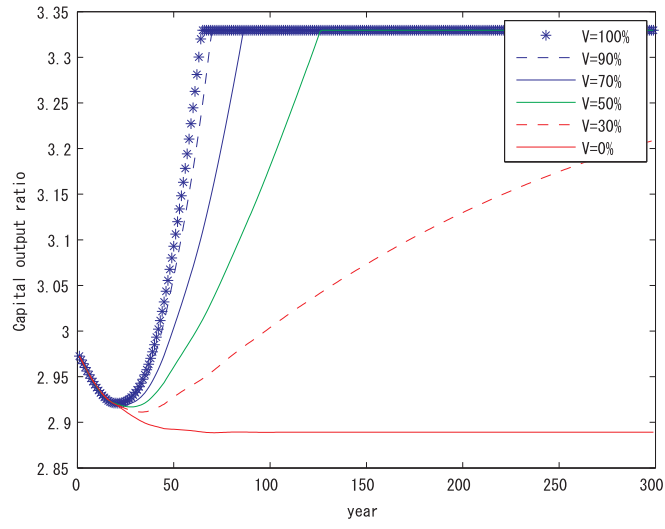


Figure A6–Capital to output ratio over time for different values of the share of the surplus used for government savings ( $V$ ). In those simulations, the number of years needed for the PITN to reach the target level is set at 80. The target capital stock is the modified golden rule level. The intergenerational discount rate for the modified golden rule is set at 3%.

Table A1 Number of birth at each age for native and immigrant in the simulation

age	number of birth of immigrant	number of birth of native
1	0.1934279	0.1743087
2	0.0520106	0.0474751
3	0.0550358	0.0505587
4	0.0569373	0.0527624
5	0.0578227	0.054141
6	0.0577998	0.0547487
7	0.0569765	0.0546402
8	0.0554604	0.0538698
9	0.0533594	0.052492
10	0.0507811	0.0505614
11	0.0478333	0.0481323
12	0.044624	0.0452592
13	0.0412605	0.0419967
14	0.0378511	0.0383992
15	0.0345031	0.034521
16	0.0313245	0.0304168
17	0.0284231	0.026141
18	0.0259064	0.0217481
19	0.0238824	0.0172925
20	0.0224589	0.0128286
21	0.0217435	0.0084112
22	0.0218439	0.0040944
23-80	0	0

Note

1. The calculator is based on Figure 3. Let  $TB(i,j)$  be the vertical axis of group  $j$  of Figure 3 where  $j$  is native or immigrant. Then, the number of births of age  $i$  of group  $j$  in the model is calculated as follows. When  $i=1$ , the number of births of age  $i$  of group  $j$  in the model is  $TB(20,j)/2$ . When  $2 \leq i \leq 22$ , the number of births is  $(TB(19+i,j)-TB(18+i,j))/2$ .

Table A2  
The effect of increasing the PITN for different inter-generational discount rates

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Row No.	years needed to reach the target PITN	inter-generational discount rate for the modified golden rule	year reaching the modified golden rule	% increase of the capital stock per efficient unit labor at the	% change of publicly provided goods per capita at the golden	% change of welfare of the cohort born at the golden rule	share of the PDV of increased publicly provided private goods in	share of the PDV of welfare gain of all natives and their descendants
1	80	0.0%	112	102.43%	35.86%	3.77%	12.23%	10.69%
2	80	0.5%	103	82.39%	35.07%	3.69%	15.39%	13.87%
3	80	1.0%	95	65.46%	33.04%	3.48%	18.10%	16.75%
4	80	1.5%	87	50.99%	30.14%	3.17%	20.26%	19.22%
5	80	2.0%	80	38.52%	26.63%	2.80%	21.80%	21.17%
6	80	2.5%	73	27.69%	22.71%	2.39%	22.64%	22.51%
7	80	3.0%	65	18.19%	18.51%	1.95%	22.71%	23.14%
8	80	3.5%	56	9.82%	14.12%	1.49%	21.86%	22.89%
9	80	4.0%	45	2.40%	9.64%	1.01%	19.80%	21.43%

**Note**

1. At the modified golden rule level, the marginal product of capital is equal to the sum of the growth rate of efficient unit of labor, the depreciation rate and the inter-generational discount rate for the modified golden rule.

## Appendix B1

Notice that in the programming problem, the objective function is concave and the constrained set is convex. Thus, if some allocation satisfies the first-order condition, it is also the solution of the programming problem. Now set up the Lagrangian function as follows:

$$\begin{aligned}
L = & \frac{1}{1+\rho} [u^o(c_1^o) + v^o(g^o, q)] \\
& + \sum_{t=1}^{\infty} \gamma_t \{ u^y(c_t^y) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho} [u^o(c_{t+1}^o) + v^o(g^o, g^{ind}) - u^*] \} \\
& + \sum_{t=1}^{\infty} \lambda_t \{ F((1+\pi_m)\alpha^* + 1 + \pi_n), (s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \\
& \quad - (c_t^o + g^o + g^{ind}) - \{(1+\pi_m)\alpha^* + 1 + \pi_n\} \times (c_t^y + s_t + a_t + g^y + g^{ind}) \}
\end{aligned}$$

The first order conditions are:

$$\begin{aligned}
c_1^o : \frac{1}{1+\rho} u^{o'}(c_1^o) = \lambda_1; c_{t+1}^o : \gamma_t \frac{1}{1+\rho} u^{o'}(c_{t+1}^o) = \lambda_{t+1}; c_t^y : \gamma_t u^{y'}(c_t^y) = \lambda_t \{(1+\pi_m)\alpha^* + 1 + \pi_n\}; \\
\gamma_t : u^y(c_t^y) + v^y(g^y, g^{ind}) + \frac{1}{1+\rho} [u^o(c_{t+1}^o) + v^o(g^o, g^{ind}) - u^*] = 0; \\
s_t + a_t : \lambda_{t+1} \left\{ \frac{\partial F}{\partial K} + 1 - \delta \right\} = \lambda_t \{(1+\pi_m)\alpha^* + 1 + \pi_n\} \\
\lambda_t : F((1+\pi_m)\alpha^* + 1 + \pi_n, s_{t-1} + a_{t-1}) + (1-\delta)(s_{t-1} + a_{t-1}) \\
- (c_t^o + g^o + q) - \{(1+\pi_m)\alpha^* + 1 + \pi_n\} \times (c_t^y + s_t + a_t + g^o + q) = 0
\end{aligned}$$

Now set the  $c_t^y, c_{t-1}^o, s_t, a_t, \lambda_t, \gamma_t$  as follows:

$$\begin{aligned}
c_t^o = c^{o*}; c_t^y = c^{y*}; s_t = s^*; a_t = 0; \lambda_1 = \frac{1}{1+\rho} u^{o'}(c^{o*}) \\
\lambda_{t+1} = \lambda_t \frac{(1+\pi_m)\alpha^* + 1 + \pi_n}{1+r^*}; \gamma_t \frac{1}{1+\rho} u^{o'}(c^{o*}) = \lambda_{t+1}
\end{aligned}$$

When we set  $c_t^y, c_{t+1}^o, s_t, a_t, \lambda_t, \gamma_t$  in this way, it clearly satisfies the first-order conditions. Thus, the initial allocation is Pareto-efficient. Q.E.D.

## Appendix B2

Using the definitions of  $\tau_{w1}$  and  $\tau_{r1}$ , the government budget surplus at period 1 is

$$\begin{aligned}
SP_1 = & w_1 l^* \sum_{j=n,m} N_1^j - w^* l^* \sum_{j=n,m} N_1^j (1 - \tau_w^*) \\
& + r_1 s^* \sum_{j=n,m} N_0^j - r^* s^* \sum_{j=n,m} N_0^j (1 - \tau_r^*) - \sum_{j=n,m} N_0^j (b^* + g^o + g^{ind}) - \sum_{j=n,m} N_1^j \times (g^y + g^{ind})
\end{aligned} \tag{1}$$



Note that  $N_1^n = N_1^n \tilde{\alpha}$  where  $\tilde{\alpha} > \alpha^*$  and that  $N_1^n$  is pre-determined where  $N_1^n = N_1^0(1 + \pi_n) + N_0^n \alpha^*(1 + \pi_m)$ .  $N_0^n = N_0^n \alpha^*$ . Thus,  $SP_1$  becomes

$$\begin{aligned} SP_1 &= w_1 l^* \sum_{j=n,m} N_1^j - w^* l_1^{*j} (1 - \tau_w^*) \{N_1^n (1 + \alpha^*) + N_1^n (\tilde{\alpha} - \alpha^*)\} \\ &\quad + r_1 s^* N_0^n (1 + \alpha^*) - r^* s^* N_0^n (1 + \alpha^*) (1 - t_r^*) - N_0^n (1 + \alpha^*) (b^* + g^o + g^{ind}) \\ &\quad - (g^y + g^{ind}) \{N_1^n (1 + \alpha^*) + N_1^n (\tilde{\alpha} - \alpha^*)\} \end{aligned} \quad (2)$$

At the steady state, as for the government budget constraint, we have

$$(\tau_w^* w^* l^* - g^y - g^{ind}) \times N_1^n \times (1 + \alpha^*) + (r^* s^* - b^* - g^o) N_0^n (1 + \alpha^*) = 0 \quad (3)$$

Thus,  $SP_1$  becomes

$$\begin{aligned} SP_1 &= w_1 l^* \sum_{j=n,m} N_1^j - w^* l_1^* \{N_1^n (1 + \alpha^*)\} - w^* l^* (1 - \tau_w^*) \{N_1^n (\tilde{\alpha} - \alpha^*)\} \\ &\quad + r_1 s^* N_0^n (1 + \alpha^*) - r^* s^* N_0^n (1 + \alpha^*) - (g^y + g^{ind}) N_1^n (\tilde{\alpha} - \alpha^*) \end{aligned}$$

From the homogeneity of the production function and Euler's theorem, we have

$$w_1 l^* \sum_{j=n,m} N_1^j + r_1 s^* N_0^n (1 + \alpha^*) = F(l^* \sum_{j=n,m} N_1^j, s^* N_0^n (1 + \alpha^*)) - \delta s^* N_0^n (1 + \alpha^*)$$

$$\text{and } w^* l^* \{N_1^n (1 + \alpha^*)\} + r^* s^* N_0^n (1 + \alpha^*) = F(l^* \{N_1^n (1 + \alpha^*)\}, s^* N_0^n (1 + \alpha^*)) - \delta^* s^* N_0^n (1 + \alpha^*)$$

Thus,  $SP_1$  becomes

$$\begin{aligned} SP_1 &= F(l^* \sum_{j=n,m} N_1^j, s^* N_0^n (1 + \alpha^*)) - \delta s^* N_0^n (1 + \alpha^*) - \{F(l_1^* \{N_1^n (1 + \alpha^*)\}, s^* N_0^n (1 + \alpha^*)) \\ &\quad - \delta^* s^* N_0^n (1 + \alpha^*)\} - w^* l^* (1 - \tau_w^*) N_1^n (\tilde{\alpha} - \alpha^*) - N_1^n (g^y + g^{ind}) (\tilde{\alpha} - \alpha^*) \end{aligned} \quad (4)$$

Note that  $N_1^n = \tilde{\alpha} N_1^n$ . Thus,

$$\begin{aligned} SP_1 &= F(l^* (N_1^n (1 + \tilde{\alpha}), s^* N_0^n (1 + \alpha^*)) - F(l^* \{N_1^n (1 + \alpha^*)\}, s^* N_0^n (1 + \alpha^*)) \\ &\quad - \{w^* l^* (1 - \tau_w^*) - (g^y + g^{ind})\} N_1^n (\tilde{\alpha} - \alpha^*) \end{aligned} \quad (5)$$

$$= \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F(N_1^n (1 + z) l^*, s^* N_0^n (1 + \alpha^*))}{\partial L} N_1^n l^* - w^* (1 - t_w) N_1^n l^* - (g^y + g^{ind}) N_1^n \right] dz \quad (6)$$

$$= N_1^n \int_{\alpha^*}^{\tilde{\alpha}} \frac{\partial F(N_1^n (1 + z) l^*, s^* N_0^n (1 + \alpha^*))}{\partial L} l^* - w^* l^* (1 - t_w) - (g^y + g^{ind}) dz \quad (7)$$

Note that  $w^* l^* (1 - t_w) = c^{y^*} + s^*$ . Thus, we have equation (25).

### Appendix B3

Note that

$$SP_2 = w_2\tau_{w2}l^*N_2^n(1 + \tilde{\alpha}) + r_2\tau_{r2}s^*N_1^n(1 + \tilde{\alpha}) - N_1^n(1 + \tilde{\alpha}) \times (b^* + g^o + g^{ind}) - N_2^n(1 + \tilde{\alpha}) \times (g^y + g^{ind}) + (1 + r_2)a_1N_1^n(1 + \tilde{\alpha}) \quad (8)$$

Using the definitions of  $\tau_{w2}$  and  $\tau_{r2}$ , we have

$$SP_2 = w_2l^*N_2^n(1 + \tilde{\alpha}) - w^*l^*N_2^n(1 + \tilde{\alpha})(1 - \tau_w^*) + r_2s^*N_1^n(1 + \tilde{\alpha}) - r^*s^*N_1^n(1 + \tilde{\alpha})(1 - \tau_r^*) - N_1^n(1 + \tilde{\alpha})(b + g^o + g^{ind}) - N_2^n(1 + \tilde{\alpha})(g^y + g^{ind}) + (1 + r_2)a_1N_1^n(1 + \tilde{\alpha}) \quad (9)$$

Thus,  $SP_2$  becomes

$$\begin{aligned} SP_2 &= w_2l^*N_2^n(1 + \tilde{\alpha}) + r_2s^*N_1^n(1 + \tilde{\alpha}) \\ &\quad - w^*l^*N_2^n(1 + \tilde{\alpha}) - r^*s^*N_1^n(1 + \tilde{\alpha}) \\ &\quad + \tau_w^*w^*l^*N_2^n(1 + \tilde{\alpha}) + \tau_r^*r^*s^*N_1^n(1 + \tilde{\alpha}) - N_1^n(1 + \tilde{\alpha})(b + g^o + g^{ind}) \\ &\quad - N_2^n(1 + \tilde{\alpha})(g^y + g^{ind}) + (1 + r_2)a_1N_1^n(1 + \tilde{\alpha}) \\ &= w_2l^*N_2^n(1 + \tilde{\alpha}) + r_2s^*N_1^n(1 + \tilde{\alpha}) + (1 + r_2)a_1N_1^n(1 + \tilde{\alpha}) \\ &\quad - w^*l^*N_2^n(1 + \tilde{\alpha}) - r^*s^*N_1^n(1 + \tilde{\alpha}) \\ &\quad + (1 + \tilde{\alpha})\{\tau_w^*w^*l^*N_2^n + \tau_r^*r^*s^*N_1^n - N_1^n(b + g^o + g^{ind}) - N_2^n(g^y + g^{ind})\} \end{aligned} \quad (10)$$

Now we calculate  $\tau_w^*w^*l^*N_2^n + \tau_r^*r^*s^*N_1^n - N_1^n(b + g^o + g^{ind}) - (g^y + g^{ind})N_2^n$ . Note that

$N_2^n = N_1^n((1 + \pi_m)\tilde{\alpha} + 1 + \pi_n) = N_1^n((1 + \pi_m)\alpha^* + 1 + \pi_n + (1 + \pi_m)\tilde{\alpha} - (1 + \pi_m)\alpha^*)$ . Thus,

$$\begin{aligned} &\tau_w^*w^*l^*N_2^n + \tau_r^*r^*s^*N_1^n - N_1^n(b + g^o + g^{ind}) - (g^y + g^{ind})N_2^n \\ &= \tau_w^*w^*l^*N_1^n((1 + \pi_m)\alpha^* + 1 + \pi_n + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)) \\ &\quad + \tau_r^*r^*s^*N_1^n - N_1^n(b + g^o + g^{ind}) \\ &\quad - (g^y + g^{ind})N_1^n((1 + \pi_m)\alpha^* + 1 + \pi_n + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)) \end{aligned} \quad (11)$$

At the initial steady state, we have

$$(\tau_w^*w^*l^* - g^y - g^{ind})(1 + \pi_n + \alpha^*(1 + \pi_m)) \times (1 + \alpha^*) + (r^*s^* - b^* - g^o)(1 + \alpha^*) = 0 \quad (12)$$

Thus, (11) becomes

$$= \tau_w^*w^*l^*N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) - (g^y + g^{ind})N_1^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \quad (13)$$

Therefore,  $SP_2$  becomes

$$\begin{aligned}
SP_2 &= w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 s^* N_1^n (1 + \tilde{\alpha}) + (1 + r_2) a_1 N_1^n (1 + \tilde{\alpha}) \\
&\quad - w^* l^* N_2^n (1 + \tilde{\alpha}) - r^* s^* N_1^n (1 + \tilde{\alpha}) \\
&\quad + (1 + \tilde{\alpha}) \{ \tau_w^* w^* l^* N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind}) N_1^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \} \quad (14)
\end{aligned}$$

Since  $N_2^n = N_1^n ((1 + \pi_n) + \tilde{\alpha}(1 + \pi_m)) = N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^* + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)\}$ ,  $SP_2$  becomes

$$\begin{aligned}
SP_2 &= w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 s^* N_1^n (1 + \tilde{\alpha}) + (1 + r_2) a_1 N_1^n (1 + \tilde{\alpha}) \\
&\quad - (1 + \tilde{\alpha}) \{ w^* l^* N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^* + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)\} + r^* s^* N_1^n \} \\
&\quad + (1 + \tilde{\alpha}) N_1^n \{ \tau_w^* w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind}) (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \}
\end{aligned}$$

Using the homogeneity of the production function, we have

$$w_2 l^* N_2^n (1 + \tilde{\alpha}) + r_2 (s^* + a_1) N_1^n (1 + \tilde{\alpha}) = F(l^* N_2^n (1 + \tilde{\alpha}), (s^* + a_1) N_1^n (1 + \tilde{\alpha})) - \delta (s^* + a_1) N_1^n (1 + \tilde{\alpha})$$

and

$$\begin{aligned}
&w^* l^* (1 + \alpha^*) N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^*\} + r^* s^* N_1^n (1 + \alpha^*) \\
&= F(l^* (1 + \alpha^*) N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^*\}, s^* N_1^n (1 + \alpha^*)) \\
&\quad - \delta s^* N_1^n (1 + \alpha^*)
\end{aligned}$$

Thus,  $SP_2$  becomes

$$\begin{aligned}
SP_2 &= F({}_2 l^* N_2^n (1 + \tilde{\alpha}), (s^* + a_1) N_1^n (1 + \tilde{\alpha})) - \delta (s^* + a_1) N_1^n (1 + \tilde{\alpha}) \\
&\quad - \frac{1 + \tilde{\alpha}}{1 + \alpha^*} \{ F(l^* (1 + \alpha^*) N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^*\}, s^* N_1^n (1 + \alpha^*)) - \delta s^* N_1^n (1 + \alpha^*) \} \\
&\quad + (1 + \tilde{\alpha}) N_1^n \{ -w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) + \tau_w^* w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind}) (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \} \\
&\quad \quad \quad + a_1 N_1^n (1 + \tilde{\alpha}) \quad (15)
\end{aligned}$$

$$\begin{aligned}
&= F({}_2 l^* N_2^n (1 + \tilde{\alpha}), (s^* + a_1) N_1^n (1 + \tilde{\alpha})) + (1 - \delta) a_1 N_1^n (1 + \tilde{\alpha}) \\
&\quad - F(l^* (1 + \tilde{\alpha}) N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^*\}, s^* N_1^n (1 + \tilde{\alpha})) \\
&\quad + (1 + \tilde{\alpha}) N_1^n \{ -w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) + \tau_w^* w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind}) (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \} \\
&\quad \quad \quad (16)
\end{aligned}$$

Subtracting and adding  $F(l^* N_2^n (1 + \tilde{\alpha}), s^* N_1^n (1 + \tilde{\alpha}))$ ,  $SP_2$  becomes

$$\begin{aligned}
SP_2 &= F(l^* N_2^n (1 + \tilde{\alpha}), (s^* + a_1) N_1^n (1 + \tilde{\alpha})) + (1 - \delta) a_1 N_1^n (1 + \tilde{\alpha}) - F(l^* N_2^n (1 + \tilde{\alpha}), s^* N_1^n (1 + \tilde{\alpha})) \\
&\quad + F(l^* N_2^n (1 + \tilde{\alpha}), s^* N_1^n (1 + \tilde{\alpha})) - F(l^* (1 + \tilde{\alpha}) N_1^n \{1 + \pi_n + (1 + \pi_m)\alpha^*\}, s^* N_1^n (1 + \tilde{\alpha})) \\
&\quad + (1 + \tilde{\alpha}) N_1^n \{ -(1 - \tau_w^*) w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind}) (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \} \quad (17)
\end{aligned}$$

Note that  $N_2^n = N_1^n((1 + \pi_m)\tilde{\alpha} + (1 + \pi_n))$ . Thus,

$$\begin{aligned}
SP_2 &= \int_{s^* N_1^n(1+\tilde{\alpha})}^{(s^*+a_1)N_1^n(1+\tilde{\alpha})} \left\{ \frac{\partial F(l^* N_2^n(1+\tilde{\alpha}), z)}{\partial K} + (1-\delta) \right\} dz \\
&+ (1+\tilde{\alpha}) N_1^n \{ F(l^* \{1 + \pi_n + (1 + \pi_m)\tilde{\alpha}\}, s^*) - F(l^* \{1 + \pi_n + (1 + \pi_m)\alpha^*\}, s^*) \\
&+ (1+\tilde{\alpha}) N_1^n \{ -(1 - \tau_w^*) w^* l^* (1 + \pi_m) (\tilde{\alpha} - \alpha^*) - (g^y + g^{ind})(1 + \pi_m) (\tilde{\alpha} - \alpha^*) \} \\
&= \int_{s^* N_1^n(1+\tilde{\alpha})}^{(s^*+a_1)N_1^n(1+\tilde{\alpha})} \left\{ \frac{\partial F(l^* N_2^n(1+\tilde{\alpha}), z)}{\partial K} + (1-\delta) \right\} dz \\
&+ (1+\tilde{\alpha}) N_1^n \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F(l^* \{1 + \pi_n + (1 + \pi_m)z\}, s^*)}{\partial L} (1 + \pi_m) l^* \right. \\
&- (1 + \pi_m) \{ (1 - \tau_w^*) w^* + (g^y + g^{ind}) \} \left. \right] dz \tag{18} \\
&= \int_{s^* N_1^n(1+\tilde{\alpha})}^{(s^*+a_1)N_1^n(1+\tilde{\alpha})} \left\{ \frac{\partial F(l^* N_2^n(1+\tilde{\alpha}), z)}{\partial K} + (1-\delta) \right\} dz \\
&+ (1+\tilde{\alpha}) N_1^n (1 + \pi_m) \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F(l^* \{1 + \pi_n + (1 + \pi_m)z\}, s^*)}{\partial L} l^* - (1 - \tau_w^*) w^* - (g^y + g^{ind}) \right] dz \tag{19}
\end{aligned}$$

Since  $w^* l^* (1 - \tau_w^*) = c^{y^*} + s^*$ , we have equation (28)

## Appendix B4

To save space, I will show that  $SP_{t+1} > 0$  for  $t = 1, 2, \dots$ . For  $t = 0$ , the same proof is applied as in the appendix A2.

We consider the same tax adjustment as in the preceding subsection.

$$(1 - \tau_{wt})w_t = (1 - \tau_w^*)w^* \text{ and } (1 - \tau_{rt})r_t = (1 - \tau_r^*)r^* \tag{20}$$

Note that

$$\begin{aligned}
SP_{t+1} &= w_{t+1} \tau_{wt+1} l^{n^*} N_{t+1}^n + \phi w_{t+1} \tau_{wt+1} l^{m^*} N_{t+1}^m + \tau_{rt+1} r_{t+1} s^{n^*} N_t^n + \tau_{rt+1} r_{t+1} s^{m^*} N_t^m \\
&- N_{t+1}^n \times (g^{y,n} + g^{ind,m}) + N_{t+1}^m \times (g^{y,m} + g^{ind,m}) \\
&+ N_t^n (b^{n^*} + g^{o,n} + g^{ind,n}) - N_t^m \times (b^{m^*} + g^{o,m} + g^{ind,m}) \\
&+ (1 + r_{t+1}) a_t (N_t^n + N_t^m) \tag{21}
\end{aligned}$$

Using the definition of  $\tau_{wt+1}$  and  $\tau_{rt+1}$ , we have

$$\begin{aligned}
\tau_{wt+1} &= \{w_{t+1} - (1 - \tau_w^*)w^*\} / w_{t+1} \\
\tau_{rt} &= \{r_{t+1} - (1 - \tau_r^*)r^*\} / r_{t+1}
\end{aligned}$$

Thus,  $SP_{t+1}$  becomes

$$\begin{aligned}
SP_{t+1} &= w_{t+1}l^{n*}N_{t+1}^n - w_{t+1}^*l^{n*}N_{t+1}^n + \tau_w^*w^*l^{m*}N_{t+1}^n + \phi w_{t+1}l^{m*}N_{t+1}^m - \phi w^*l^{m*}N_{t+1}^m + \phi\tau_w^*w^*l^{m*}N_{t+1}^m \\
&+ r_{t+1}s^{n*}N_t^n - r^*s^{n*}N_t^n + \tau_r^*r^*s^{n*}N_t^n + \\
&+ r_{t+1}s^{m*}N_t^m - r^*s^{m*}N_t^m + \tau_r^*r^*s^{m*}N_t^m \\
&- N_{t+1}^n \times (g^{y,n} + g^{ind,n}) - N_{t+1}^m \times (g^{y,m} + g^{ind,m}) \\
&- N_t^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_t^m \times (b^{m*} + g^{o,m} + g^{ind,m}) \\
&+ (1 + r_{t+1})a_t(N_t^n + N_t^m)
\end{aligned} \tag{22}$$

Now we need to calculate the following term:

$$\begin{aligned}
&\tau_w^*w^*l^{n*}N_{t+1}^n + \phi\tau_w^*w^*l^{m*}N_{t+1}^m + \tau_r^*r^*s^{n*}N_t^n + \tau_r^*r^*s^{m*}N_t^m \\
&- N_{t+1}^n \times (g^{y,n} + g^{ind,n}) - N_{t+1}^m \times (g^{y,m} + g^{ind,m}) \\
&- N_t^n (b^{n*} + g^{o,n} + g^{ind,n}) - N_t^m \times (b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned} \tag{23}$$

Note that  $N_{t+1}^n = N_t^n(1 + \pi_n + \tilde{\alpha}(1 + \pi_m))$  and  $N_{t+1}^m = N_{t+1}^n\tilde{\alpha}$ . Also at the initial steady state,  $N_{t+1}^n = N_t^n(1 + \pi_n + \alpha^*(1 + \pi_m))$  and  $N_{t+1}^m = N_{t+1}^n\alpha^*$ . On the other hand, at the initial steady state, the government budget constraint implies

$$\begin{aligned}
&\tau_w^*w^*N_t^n(1 + \pi_n + \alpha^*(1 + \pi_m))l^{n*} + \tau_w^*\phi w^*N_t^n(1 + \pi_n + \alpha^*(1 + \pi_m))l^{m*}\alpha^* \\
&+ \tau_r^*r^*s^{n*}N_t^n + \tau_r^*r^*s^{m*}N_t^m \\
&- N_t^n(1 + \pi_n + \alpha^*(1 + \pi_m)) \times (g^{y,n} + g^{ind,n}) + N_t^n(1 + \pi_n + \alpha^*(1 + \pi_m))\alpha^*(g^{y,m} + g^{ind,m}) \\
&- N_t^n(b^{n*} + g^{o,n} + g^{ind,n}) - N_t^n\alpha^*(b^{m*} + g^{o,m} + g^{ind,m})
\end{aligned} \tag{24}$$

Thus, (23) becomes

$$\begin{aligned}
&\tau_w^*w^*l^{n*}N_{t+1}^n + \phi\tau_w^*w^*l^{m*}N_{t+1}^m + \tau_r^*r^*s^{n*}N_t^n + \tau_r^*r^*s^{m*}N_t^m \\
&- N_{t+1}^n \times (g^{y,n} + g^{ind,n}) - N_{t+1}^m \times (g^{y,m} + g^{ind,m}) \\
&- N_t^n(b^{n*} + g^{o,n} + g^{ind,n}) - N_t^m \times (b^{m*} + g^{o,m} + g^{ind,m}) \\
&= \tau_w^*w^*l^{n*}N_t^n(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{25}$$

$$\begin{aligned}
&+ \phi\tau_w^*w^*l^{m*}N_t^n\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&+ \tau_r^*r^*s^{m*}N_t^n(\tilde{\alpha} - \alpha^*) \\
&- N_t^n(1 + \pi_m)[g^{y,n} + g^{ind,n}](\tilde{\alpha} - \alpha^*) \\
&- N_t^n(1 + \pi_m)[g^{y,n} + g^{ind,n}]\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&- N_t^n(b^{m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{26}$$

where  $J(z) = (1 + \pi_n + z(1 + \pi_m))z$

Therefore,  $SP_{t+1}$  becomes

$$\begin{aligned}
SP_{t+1} &= w_{t+1}l^{n*}N_{t+1}^n + \phi w_{t+1}l^{m*}N_{t+1}^m \\
&\quad - w^*l^{n*}N_{t+1}^n - \phi w^*l^{m*}N_{t+1}^m \\
&\quad + r_{t+1}s^{n*}N_t^n + r_{t+1}s^{m*}N_t^m + (1 + r_{t+1})a_t(N_t^n + N_t^m) \\
&\quad - r^*s^{n*}N_t^n - r^*s^{m*}N_t^m \\
&\quad + \tau_w^*w^*l^{n*}N_t^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&\quad + \phi\tau_w^*w^*l^{m*}N_t^m\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad + \tau_r^*r^*s^{m*}N_t^m(\tilde{\alpha} - \alpha^*) \\
&\quad - N_t^n(1 + \pi_m)(g^{y,n} + g^{ind,n})(\tilde{\alpha} - \alpha^*) \\
&\quad - N_t^n(g^{y,m} + g^{ind,m})\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad - N_t^n(b^{m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{27}$$

Note that

$$\begin{aligned}
&w^*l^{n*}N_{t+1}^n + \phi w^*l^{m*}N_{t+1}^m + r^*s^{n*}N_t^n + r^*s^{m*}N_t^m \\
&= w^*l^{n*}N_t^n\{1 + \pi_n + \alpha^*(1 + \pi_m) + (1 + \pi_m)(\tilde{\alpha} - \alpha^*)\} \\
&\quad + \phi w^*l^{m*}N_t^m\{J(\alpha^*) + J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad + r^*s^{n*}N_t^n + r^*s^{m*}N_t^m(\alpha^* + \tilde{\alpha} - \alpha^*) \\
&\quad + \delta\{s^{n*}N_t^n + s^{m*}N_t^m\} - \delta\{s^{n*}N_t^n + s^{m*}N_t^m\} \\
&= F((1 + \pi_n + (1 + \pi_m)\alpha^*)l^{n*} + \phi(1 + \pi_n + (1 + \pi_m)\alpha^*)\alpha^*l^{m*}, s^{n*}N_t^n + s^{m*}N_t^m\alpha^*) \\
&\quad - \delta\{s^{n*}N_t^n + s^{m*}N_t^m\} \\
&\quad + w^*l^{n*}N_t^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&\quad + \phi w^*l^{m*}N_t^m\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad + r^*s^{m*}N_t^m(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{28}$$

Using the homogeneity of the production function and (28),  $SP_{t+1}$  becomes as follows:

$$\begin{aligned}
SP_{t+1} &= F(N_{t+1}^n l^{n*} + \phi N_{t+1}^m l^{m*}, s^{n*} N_t^n + s^{m*} N_t^m + a_t(N_t^n + N_t^m)) \\
&\quad - \delta\{s^{n*} N_t^n + s^{m*} N_t^m + a_t(N_t^n + N_t^m)\} + a_t(N_t^n + N_t^m) \\
&\quad - F((1 + \pi_n + (1 + \pi_m)\alpha^*)l^{n*} + \phi(1 + \pi_n + (1 + \pi_m)\alpha^*)\alpha^* l^{m*}, s^{n*} N_t^n + s^{m*} N_t^m \alpha^*) \\
&\quad + \delta\{s^{n*} N_t^n + s^{m*} N_t^m \alpha^*\} \\
&\quad - w^* l^{n*} (1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&\quad - \phi w^* l^{m*} \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad - r^* s^{m*} N_t^n (\tilde{\alpha} - \alpha^*) \\
&\quad + \tau_w^* w^* l^{n*} N_t^n (1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&\quad + \phi \tau_w^* w^* l^{m*} N_t^n \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad + \tau_r^* r^* s^{m*} N_t^n (\tilde{\alpha} - \alpha^*) \\
&\quad - N_t^n (1 + \pi_m)(g^{y,n} + g^{ind,n})(\tilde{\alpha} - \alpha^*) \\
&\quad - N_t^n (g^{y,m} + g^{ind,m})\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad - N_t^n (b^{m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{29}$$

By subtracting and adding  $F(N_{t+1}^n l^{n*} + \phi N_{t+1}^m l^{m*}, s^{n*} N_t^n + s^{m*} N_t^m)$ ,  $SP_{t+1}$  becomes

$$\begin{aligned}
SP_{t+1} &= \int_{s^{n*} N_t^n + s^{m*} N_t^m}^{s^{n*} N_t^n + s^{m*} N_t^m + a_t(N_t^n + N_t^m)} \left\{ \frac{\partial F(N_{t+1}^n l^{n*} + \phi N_{t+1}^m l^{m*}, z)}{\partial K} + (1 - \delta) \right\} dz \\
&\quad + F(N_{t+1}^n l^n + \phi N_{t+1}^m \tilde{\alpha} l^m, s^{n*} N_t^n + s^{m*} N_t^m \tilde{\alpha}) \\
&\quad - F(N_t^n (1 + \pi_n + \alpha^*(1 + \pi_m))l^{n*} + \phi N_t^m (1 + \pi_n + \alpha^*(1 + \pi_m))\alpha^* l^{m*}, s^{n*} N_t^n + s^{m*} N_t^m \alpha^*) \\
&\quad - \delta s^{m*} N_t^n (\tilde{\alpha} - \alpha^*) \\
&\quad - (1 - \tau_w^*) w^* l^{n*} N_t^n (1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&\quad - (1 - \tau_w^*) \phi w^* l^{m*} N_t^n \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad - (1 - \tau_r^*) r^* s^{m*} N_t^n (\tilde{\alpha} - \alpha^*) \\
&\quad - N_t^n (1 + \pi_m)(g^{y,n} + g^{ind,n})(\tilde{\alpha} - \alpha^*) \\
&\quad - N_t^n (g^{y,m} + g^{ind,m})\{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&\quad - N_t^n (b^{m*} + g^{o,m} + g^{ind,m})(\tilde{\alpha} - \alpha^*)
\end{aligned} \tag{30}$$

Let  $c^{o,m*}$  be the consumption of old immigrants at the initial steady state. Then,  $c^{o,m*} = b^m + (1 + (1 - \tau_r^*)r^*)s^{m*}$ . Thus,  $SP_{t+1}$  becomes

$$\begin{aligned}
SP_{t+1} &= \int_{s^{n^*}N_t^n + s^{m^*}N_t^m}^{s^{n^*}N_t^n + s^{m^*}N_t^m + a_t(N_t^n + N_t^m)} \left\{ \frac{\partial F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m l^{m^*}, z)}{\partial K} + (1 - \delta) \right\} dz \\
&+ F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m \tilde{\alpha} l^{m^*}, s^{n^*}N_t^n + s^{m^*}N_t^m \tilde{\alpha}) \\
&- F(N_t^n (1 + \pi_n + \alpha^* (1 + \pi_m)) l^{n^*} + \phi N_t^m (1 + \pi_n + \alpha^* (1 + \pi_m)) \alpha^* l^{m^*}, s^{n^*}N_t^n + s^{m^*}N_t^m \alpha^*) \\
&- (1 - \delta) s^{m^*} N_t^n (\tilde{\alpha} - \alpha^*) \\
&- (1 - \tau_w^*) w^* l^{n^*} N_t^n (1 + \pi_m) (\tilde{\alpha} - \alpha^*) \\
&- (1 - \tau_w^*) \phi w^* l^{m^*} N_t^m \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&- (1 - \tau_r^*) r^* s^{m^*} N_t^n (\tilde{\alpha} - \alpha^*) \\
&- N_t^n (1 + \pi_m) (g^{y,n} + g^{ind,n}) (\tilde{\alpha} - \alpha^*) \\
&- N_t^m (g^{y,m} + g^{ind,m}) \{J(\tilde{\alpha}) - J(\alpha^*)\} \\
&- N_t^n (c^{o,m^*} + g^{o,m} + g^{ind,m}) (\tilde{\alpha} - \alpha^*) \tag{31}
\end{aligned}$$

Note that the second line and the third line of the above equation can be transformed as follows:

$$\begin{aligned}
&F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m \tilde{\alpha} l^{m^*}, s^{n^*}N_t^n + s^{m^*}N_t^m \tilde{\alpha}) \\
&- F(N_t^n (1 + \pi_n + \alpha^* (1 + \pi_m)) l^{n^*} + \phi N_t^m (1 + \pi_n + \alpha^* (1 + \pi_m)) \alpha^* l^{m^*}, s^{n^*}N_t^n + s^{m^*}N_t^m \alpha^*) \\
&= F(N_t^n (1 + \pi_n + \tilde{\alpha} (1 + \pi_m)) \times (l^{n^*} + \phi \tilde{\alpha} l^{m^*}), s^{n^*}N_t^n + s^{m^*}N_t^m \tilde{\alpha}) \\
&- F(N_t^n (1 + \pi_n + \alpha^* (1 + \pi_m)) l^{n^*} + \phi N_t^m (1 + \pi_n + \alpha^* (1 + \pi_m)) \alpha^* l^{m^*}, s^{n^*}N_t^n + s^{m^*}N_t^m \alpha^*) \\
&= \int_{\alpha^*}^{\tilde{\alpha}} \left\{ \frac{\partial F(N_t^n \{(1 + \pi_n + z(1 + \pi_m)) l^{n^*} + \phi J(z) l^{m^*}\}, N_t^m \{s^{n^*} + s^{m^*} z\})}{\partial L} [N_t^n (1 + \pi_m) l^{n^*} + N_t^m \phi J'(z) l^{m^*}] \right. \\
&+ \left. \frac{\partial F(N_t^n \{(1 + \pi_n + z(1 + \pi_m)) l^{n^*} + \phi J(z) l^{m^*}\}, N_t^m \{s^{n^*} + s^{m^*} z\})}{\partial K} s^{m^*} N_t^m \right\} dz
\end{aligned}$$

Using the homogeneity of  $\partial F/\partial K$  and  $\partial F/\partial L$ , we have



$$\begin{aligned}
SP_{t+1} &= \int_{s^{n^*}N_t^n + s^{m^*}N_t^m + a_t(N_t^n + N_t^m)}^{s^{n^*}N_{t+1}^{n^*} + s^{m^*}N_{t+1}^{m^*} + a_t(N_t^n + N_t^m)} \left\{ \frac{\partial F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m l^{m^*}, z)}{\partial K} + (1 - \delta) \right\} dz \\
&+ \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F((1 + \pi_n + z(1 + \pi_m))l^{n^*} + \phi J(z)l^{m^*}, s^{n^*} + s^{m^*}z)}{\partial K} + (1 - \delta) \right] s^{m^*} N_t^n dz \\
&+ \int_{\alpha^*}^{\tilde{\alpha}} \frac{\partial F((1 + \pi_n + z(1 + \pi_m))l^{n^*} + \phi J(z)l^{m^*}, s^{n^*} + s^{m^*}\alpha^*)}{\partial L} [N_t^n(1 + \pi_m)l^{n^*} + N_t^m \phi J'(z)l^{m^*}] dz \\
&\quad - \int_{\alpha^*}^{\tilde{\alpha}} [(1 - \tau_w^*)w^*l^{n^*}N_t^n(1 + \pi_m)(\tilde{\alpha} - \alpha^*) \\
&\quad \quad + (1 - \tau_w^*)\phi w^*l^{m^*}N_t^m J'(z) \\
&\quad \quad + (1 - \tau_r^*)r^*s^{m^*}N_t^m \\
&\quad \quad + N_t^n(1 + \pi_m)(g^{y,n} + g^{ind,n}) \\
&\quad \quad + N_t^m(g^{y,m} + g^{ind,m})J'(z) \\
&\quad \quad + N_t^m(c^{o,m^*} + g^{o,m} + g^{ind,m})] dz \tag{32}
\end{aligned}$$

Note that  $J'(z) = (1 + \pi_m)z + (1 + \pi_n + (1 + \pi_m)z)$ . By rearranging the above equation, we have

$$\begin{aligned}
SP_{t+1} &= \int_{s^{n^*}N_t^n + s^{m^*}N_t^m + a_t(N_t^n + N_t^m)}^{s^{n^*}N_{t+1}^{n^*} + s^{m^*}N_{t+1}^{m^*} + a_t(N_t^n + N_t^m)} \left\{ \frac{\partial F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m l^{m^*}, z)}{\partial K} + (1 - \delta) \right\} dz \\
&+ (1 + \pi_m)N_t^n \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F((1 + \pi_n + z(1 + \pi_m))l^{n^*} + \phi J(z)l^{m^*}, s^{n^*} + s^{m^*}z)}{\partial L} l^{n^*} \right. \\
&\quad \left. - (1 - \tau_w^*)w^*l^{n^*} - g^{y,n} - g^{ind,n} \right] dz \\
&+ (1 + \pi_m)N_t^m \int_{\alpha^*}^{\tilde{\alpha}} z \left[ \frac{\partial F((1 + \pi_n + z(1 + \pi_m))l^{n^*} + \phi J(z)l^{m^*}, s^{n^*} + s^{m^*}z)}{\partial L} l^{m^*} \right. \\
&\quad \left. - \phi(1 - \tau_w^*)w^*l^{m^*} - g^{y,m} - g^{ind,m} \right] dz \\
&+ N_t^n \int_{\alpha^*}^{\tilde{\alpha}} \left\{ (1 + \pi_n + (1 + \pi_m)z) \times \left[ \frac{\partial F((1 + \pi_n + z(1 + \pi_m))l^{n^*} + \phi J(z)l^{m^*}, s^{n^*} + s^{m^*}z)}{\partial L} \phi l^{m^*} \right. \right. \\
&\quad \left. \left. - \phi(1 - \tau_w^*)w^*l^{m^*} - g^{y,m} - g^{ind,m} \right] - (c^{o,m^*} + g^{o,m} + g^{ind,m}) \right\} dz \\
&+ N_t^m \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F((1 + \pi_n + z(1 + \pi_m))l^{n^*} + \phi J(z)l^{m^*}, s^{n^*} + s^{m^*}z)}{\partial K} + (1 - \delta) \right] s^{m^*} dz \tag{33}
\end{aligned}$$

The first line is positive from the condition related with the golden rule. The second and third lines are the MPL condition. The fourth and fifth lines are the redistribution effect. The fourth line is the difference between what immigrants contribute minus the resources used for immigrants. Note that when natives and immigrants are identical, we can show that (33) becomes (28) by using the government budget constraint.

## Appendix B5

From the discussion of A4, the government budget surplus at period  $t + 1$  can be rewritten as

$$\begin{aligned}
SP_{t+1} &= \int_{s^{n^*}N_t^n + s^{m^*}N_t^m}^{s^{n^*}N_t^n + s^{m^*}N_t^m + a_t(N_t^n + N_t^m)} \left\{ \frac{\partial F(N_{t+1}^n l^n + \phi N_{t+1}^m l^m, z)}{\partial K} + (1 - \delta) \right\} dz \\
&+ (1 + \pi_m) N_t^n \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F(N_t^n (1 + \pi_n + z(1 + \pi_m)) l^{n^*} + \phi J(z) l^{m^*}, s^{n^*} N_t^n + s^{m^*} N_t^n \alpha^*)}{\partial L} l^{m^*} \right. \\
&\quad \left. - (1 - \tau_w^*) w^* l^{n^*} - g^{y,n} - g^{ind,n} \right] dz \\
&+ (1 + \pi_m) N_t^n \int_{\alpha^*}^{\tilde{\alpha}} z \left[ \frac{\partial F(N_t^n (1 + \pi_n + z(1 + \pi_m)) l^{n^*} + \phi J(z) l^{m^*}, s^{n^*} N_t^n + s^{m^*} N_t^n \alpha^*)}{\partial L} l^{m^*} \right. \\
&\quad \left. - \phi(1 - \tau_w^*) w^* l^{m^*} - g^{y,m} - g^{ind,m} \right] dz \\
&+ N_t^n \int_{\alpha^*}^{\tilde{\alpha}} \left\{ (1 + \pi_n + (1 + \pi_m)z) \times \left[ \frac{\partial F(N_t^n (1 + \pi_n + z(1 + \pi_m)) l^{n^*} + \phi J(z) l^{m^*}, s^{n^*} N_t^n + s^{m^*} N_t^n \alpha^*)}{\partial L} \phi l^{m^*} \right. \right. \\
&\quad \left. \left. - \phi(1 - \tau_w^*) w^* l^{m^*} - g^{y,m} - g^{ind,m} \right] - (c^{o,m^*} + g^{o,m} + g^{ind,m}) \right\} dz \\
&+ \int_{\alpha^*}^{\tilde{\alpha}} \left[ \frac{\partial F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m \tilde{\alpha} l^{m^*}, s^{n^*} N_t^n + s^{m^*} N_t^n z)}{\partial K} + (1 - \delta) \right] s^{m^*} N_t^n dz
\end{aligned}$$

Since  $SP_1 > 0$ , the government can accumulate savings. Thus,  $a_1 > 0$ . For  $a_t$  ( $t \geq 2$ ), assume that the government uses the surplus that arises from the first term. This implies

$$a_{t+1} = \frac{1}{N_{t+1}^n + N_{t+1}^m} \int_{s^{n^*}N_t^n + s^{m^*}N_t^m}^{s^{n^*}N_t^n + s^{m^*}N_t^m + a_t(N_t^n + N_t^m)} \left\{ \frac{\partial F(N_{t+1}^n l^{n^*} + \phi N_{t+1}^m l^{m^*}, z)}{\partial K} + (1 - \delta) \right\} dz$$

Then, the proof is the same as for the proposition 3.

## Appendix B6

$\Pi_{q=1}^i p_q$  is the probability that a native is still alive at age  $i$  given that an agent is alive at age 1 and  $\Pi_{q=1}^i p_q \hat{p}_q$  is the probability that an immigrant is alive and stays in the host country at the age  $i$ . Then,  $N_t^{in} = N_{t-(i-1)}^{1n} \times \Pi_{q=1}^i p_q$  and  $N_t^{im} = N_{t-(i-1)}^{1m} \times \Pi_{q=1}^i p_q \hat{p}_i$ . Thus,  $N_{t+20}^{1,n}/(1-d)$  becomes as follows:

$$\frac{N_{t+20}^{1,n}}{1-d} = \sum_{i=1}^{80} \eta^{i,m} N_{t-(i-1)}^{1,m} \times \Pi_{q=1}^i p_q \hat{p}_q \Pi_{x=1}^{20} \hat{p}_{i+x} + \sum_{i=1}^{80} N_{t-(i-1)}^{1,n} \eta^{i,n} \times \Pi_{q=1}^i p_q \quad (34)$$

Note that,  $N_{t+20}^{1,n} = N_t^{1,n} (1 + \sigma)^{20}$  and  $N_{t-(i-1)}^{1,j} = N_t^{1,j} / (1 + \sigma)^{i-1}$  for  $j = i, m$ . Thus, (34) becomes

$$\frac{N_t^{1,n} (1 + \sigma)^{20}}{1-d} = \sum_{i=1}^{80} \frac{N_t^{1,m}}{(1 + \sigma)^{i-1}} \eta^{i,m} \times \Pi_{q=1}^i p_q \hat{p}_q \Pi_{x=1}^{20} \hat{p}_{i+x} + \sum_{i=1}^{80} \frac{N_t^{1,n}}{(1 + \sigma)^{i-1}} \eta^{i,n} \times \Pi_{q=1}^i p_q$$

Let the steady state value of the age 1 immigrant native ratio,  $N_t^{1,m}/N_t^{1,n}$  be  $f$ . Solving for  $f$ , we have

$$f = \frac{\frac{1}{1-d} - \sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta^{i,n} \times \prod_{q=1}^i p_q}{\sum_{i=1}^{80} \frac{1}{(1+\sigma)^{i+19}} \eta^{i,m} \times \prod_{q=1}^i p_q \hat{p}_q \prod_{x=1}^{20} \hat{p}_{i+x}} \quad (35)$$

Thus, the ratio of immigrants to natives at the steady state becomes (40).